Quality Growth: From Process to Product Innovation along the Path of Development*

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Abstract

We propose a demand-driven growth theory where process innovations and product innovations fulfil sequential roles along the growth path. Process innovations must initially set the economy on a positive growth path. However, process innovations alone cannot fuel growth forever, as their benefits display an inherent tendency to wane. Product innovations are therefore also needed for the economy to keep growing in the long run. When the economy fails to switch from a growth regime steered by process innovation to one driven by product innovation, R&D effort and growth will eventually come to a halt. However, when the switch to a product innovation growth regime does take place, a virtuous circle gets ignited. This happens because product innovation effort not only keeps growth alive when incentives to undertake process innovation diminish, but it also regenerates profit prospects from further process innovation effort.

Keywords: Endogenous Growth, Process and Product Innovation, Nonhomothetic Preferences, Quality Ladders.

JEL Classifications: O30, O31, O41

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1 Introduction

Process and product innovations are two key determinants behind sustained economic growth. Process innovations introduce technological improvements that allow an expansion in the quantity of goods that an economy can deliver. Product innovations foster growth instead by bringing to the market goods of higher quality than those previously available. This paper presents a demand-driven growth theory in which both types of innovations fulfil crucial roles, and where their respective roles display a specific sequential timing. Our theory shows that process innovations must precede product innovations along the path of development. Yet, while process innovations can initially set the economy on a positive growth path, they cannot sustain rising incomes perpetually. Long-lasting growth requires that the economy is also able to start generating product innovations at some point. The reason for this is that, without the help of quality-upgrading innovations, the incentives to invest in process innovations will eventually start to wane as physical production increasingly expands.

The model features an economy with a vertically differentiated good, available in a number of (vertically ordered) quality levels. All the quality levels are produced with technologies that use labor as their sole input. Both labor productivity and the degree of vertical differentiation are endogenous to the model. Labor productivity increases as a result of process innovations. In particular, process innovations lower the costs of production (in terms of hours of labor), leading to an increase in the physical quantities that may be produced with a given amount of labor. Product innovations instead allow the introduction of better quality versions of the vertically differentiated good.

Innovations are the outcome of purposeful research and development effort. Hence, investment in process and in product innovations will be the endogenous response to the potential profit associated to each of them. The underlying force leading to their different timings along the growth path stems from our demand side. Individuals exhibit nonhomothetic preferences along the quality dimension. In particular, their willingness to pay for varieties of higher quality increases as their incomes rise. An implication of this is that product innovations tend to become increasingly profitable along the growth path, since product innovators can charge higher mark-ups when they face richer consumers. However, our nonhomothetic demand structure entails also a flip side. At early stages of development, the economy must rely on process innovations as the source of income growth. This is because the low willingness to pay for quality by consumers with low incomes stifles profit opportunities for product innovators.

Our theory then shows that at early stages of development process innovation must become the leading actor. Product innovation takes over a more prominent role instead in more mature economies. Furthermore, such transition from process to product innovation effort proves essential for sustaining growth in our model. In a context where individuals display decreasing marginal utility on physical consumption, process innovations bring about two opposing dynamic forces. On the one hand, they drive the marginal utility of consumption down. Thus, the prospects of *future* profits from process innovation are endogenously dampened by *current* process innovation efforts. On the other hand, the higher quantity of consumption allowed by process innovations is exactly what spurs profit prospects from product innovation effort. The tension between these two countervailing forces means that an economy may or may not succeed in eventually switching from a growth path steered by process innovation to one steered by product innovation. When it fails to do so, growth will eventually come to a halt due to the negative effect of decreasing marginal utility on process innovation profits.

The switch to a product innovation growth regime can in turn ignite a virtuous circle with further process innovation down the road. The incapacity of cost-cutting innovations to spur growth perpetually lies in that a continuous expansion in quantity of production must struggle against the decreasing marginal utility of (physical) consumption that it simultaneously leads to. This struggle makes it increasingly hard to keep profit prospects from process innovation high enough to sustain it forever. Product innovations are able to relax this inherent tension. In particular, by raising the intrinsic quality of goods offered in the economy, product innovations make the decreasing marginal utility of (physical) consumption less pressing, and thereby regenerate profit prospects from further process innovation effort.

Our theory yields thus a model where long-run growth stems from a positive feedback loop between cost-cutting and quality-enhancing innovations. The endogenous growth literature has produced several models where growth results from the interplay of different types of innovation effort, such as general purpose technologies and sector-specific complementary inputs [e.g., Bresnahan and Trajtenberg (1995), Helpman and Trajtenberg (1998a, 1998b)], fundamental research/breakthroughs and secondary development [e.g., Jovanovic and Rob (1990), Aghion and Howitt (1996), Redding (2002)], invention and learning-by-doing [e.g., Young (1993), Stein (1997)]. All these models share a common trait: there exists one fundamental source of long-run growth that interacts with a transitional and bounded source of growth linked to the fundamental technology. Our model departs from the notion of fundamental and secondary sources of technical change. We look at the specific case of process and product innovations as *potentially* independent sources of technical change, but *ultimately* unable to sustain unbounded growth without one another. This, in turn, leads to a specific sequential role by each type of innovation along the growth path, which is the core message of this paper.

The number of articles dealing separately with either process or product innovation effort in the endogenous growth literature is huge. Yet, it is hard to find models where both are explicitly involved together in steering the economy along the growth path, while at the same time playing a distinctive role as growth engines.[1] One prominent example is Foellmi, Wuergler and Zweimüller (2014), who build a growth model with non-homothetic preferences where firms must choose between product innovations to introduce new luxury goods to be consumed only by the rich, or process innovations that turn luxuries into mass consumption goods also available to the poor. Their model depicts situations where this type of product cycle arises as the optimal behavior by firms, and use it to explain how some new goods first introduced during the 20th century have later on become available as mass consumption goods (e.g., automobiles, refrigerators, etc.).[2] The main focus of our model is somewhat different, as it studies how the interplay between process and product innovation efforts can sustain a continuous increase in incomes in the long run, and how the preeminence of each type of innovation shifts along the growth path. In that sense, our model is mostly concerned with how an economy may keep growing *beyond* a mass consumption economy, in a context where rising incomes increasingly tilt consumer preferences towards quality expansion (and away from quantity expansion).[3]

Besides Foellmi *et al* (2014), a few other articles have mingled together vertical and horizontal innovations; see, e.g., Peretto (1998), Peretto and Connolly (2007), Sorger (2011), Chu, Cozzi and Galli (2012), Akcigit and Kerr (2018), Flach and Irlacher (2018). These articles, however, have all remained within standard homothetic frameworks, where trade-offs and in-

[1] Models where growth is the result of the of new technologies that allow an increase in physical production (i.e., process innovations) can be found in: Shleifer (1986), Aghion and Howitt (1992), Jones (1995), Kortum (1997). Examples of models where growth is driven by the introduction of final goods of higher quality than before (i.e., product innovations) are: Segerstrom et al. (1990), Grossman and Helpman (1991a, 1991b), Stokey (1991), Segerstrom (1998). A third type of innovation, which is neglected by our model, is that one that leads to a horizontal expansion in the variety of goods, as in Judd (1985), Romer (1990), Grossman and Helpman (1991c, Ch. 3), Young (1993). We relegate to the concluding section a brief discussion on the possible effects of introducing variety-expanding innovations within the context of our model.

[2] Matsuyama (2002) also studies an endogenous growth model where goods initially affordable to the rich become gradually mass consumption goods affordable to all individuals. In his model, however, technological change is not the result of purposeful R&D effort, but it arises because of industry-specific learning-by-doing.

[3] Foellmi and Zweimüller (2006) also present a demand-driven endogenous growth model where individuals display non-homothetic preferences. Their model differs from ours substantially, with two key differences: i) in their model there is no quality differentiation (their nonhomotheticities are the result of hierarchical preferences with a horizontal continuum of goods); ii) their model features only cost-cutting innovations, which is combined with a setup cost that must be incurred to open new sectors/product lines.

teractions faced by innovators when choosing between process and product innovations are disconnected from changes of consumer behavior along the growth path. As a result, these models remain silent about the needed transition from process to product innovation in order to keep richer consumers' demands continuously unsatiated.

A key aspect behind our demand-driven growth model is therefore the nonhomotheticity of preferences along the quality dimension (i.e., the notion that willingness to pay for quality upgrading rises with income). This is in fact a property of the preference structure that has been previously incorporated in several trade models [e.g., Flam and Helpman (1987), Murphy and Shleifer (1997), Fajgelbaum, Grossman and Helpman (2011, 2015), Jaimovich and Merella (2012, 2015)], and it has also been widely supported both by household-level data based on consumer surveys [e.g., Bils and Klenow (2001)] and bilateral trade flows data [e.g., Hallak (2006) and Choi *et al.* (2009)]. In our model, quality upgrading arises endogenously as the result of firms' effort to cater to consumers with rising incomes.[4] In addition to such quality-upgrading effort in response to rising incomes, our theory also predicts a gradual shift from process innovation to product innovation along the growth path. This required growth transition echoes the current policy discussions summarized in the 'Made in China 2025' initiative, which aims to bolster quality of production in the Chinese manufacturing sector.[5] Moreover, such dynamic transition seems also to be in line with the R&D behavior by firms in countries with different incomes, as reported in Table 1.

Table 1 displays a series of simple correlations between the innovation intensity by type of innovation at the firm level, and the income per head of the country where the firm is located.[6] The data on innovation intensity by type of innovation (process innovation vs. product innovation) is taken from the Eurostat *Community Innovation Survey* (CIS), which collects information about the innovation activity by enterprises. The CIS asks the surveyed firms if

[4]This is also a phenomenon that has received support by a growing strand of empirical papers; e.g. Verhoogen (2008), Brambilla *et al.* (2012), Manova and Zhang (2012), Bas and Strauss-Kahn (2015), Flach (2016).

[5]A quick summary of the main objectives of the 'Made in China 2025' initiative can be found on https://www.csis.org/analysis/made-china-2025, where explicit reference is made to the fact one of the guiding principles 'are to have manufacturing be innovation-driven, [and] emphasize quality over quantity [...]'. See also the explicit reference by China's Premier Li Keqiang in http://www.china.org.cn/business/2015-03/30/content_35192417.htm to the fact that '[China] will redouble its efforts to upgrade from a manufacturer of quantity to one of quality'. One of the stated reasons behind the 'Made in China 2025' initiative is to be able to meet the higher demand for quality and high-end products by the growing middle-income Chinese population.

[6]Table B.2 in Appendix B complements Table 1 by including, alongside GDP per capita, a few additional regressors that may heterogeneously influence different types of innovation effort, namely: total GDP, the level of financial development, and the stock of human capital.

they have introduced innovations during the previous two years, and whether these innovations pertain to process innovation or product innovation (or both).[7]

The first column in Table 1 correlates the share of firms in each country that have introduced either process innovations or product innovations (or both) during 2004-2006 on the income per head of each country. The correlation is positive and highly significant, suggesting that firms in richer economies tend to introduce more innovations (of any type). The next two columns assess the correlation separately for each type of innovation. The second column considers as the dependent variable *only* those firms that have introduced *process innovations*, while the third column looks *only* at those firms that have introduced *product innovations*. In both cases, we see again positive and significant correlations. Yet, the magnitude of the correlation displayed in the third column is significantly stronger than in the second column. Figure 1 provides a visual description the regressions in the second and third columns showing their respective scatter plots. (The results are robust to excluding Luxembourg from the set of countries.) Finally, column four uses as dependent variable the *ratio* of firms that introduced product innovations over those reporting process innovations. This regression reveals again that the share of firms doing product innovation over those doing process innovations is greater in richer countries.[8]

The correlations in Table 1 (and also those in Table B.1 and B.2 in Appendix B) point out towards an interesting pattern: while R&D activities are larger in richer countries than in poorer ones, this behavior is more pronounced for product innovations than for process innovations. These correlations motivate our demand-driven endogenous growth model, where the importance of quality-upgrading innovations as a growth engine tends to rise relative to that of cost-cutting innovations as individuals get richer along the growth path.[9]

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[7] The CIS defines: *i*) 'A *process innovation* is the implementation of a new or significantly improved production process, distribution method, or support activity for your goods or services. [...] [It] excludes purely organisational innovations.'; *ii*) 'A *product innovation* is the introduction of a new good or service or a significantly improved good or service with respect to its capabilities, such as improved software, user friendliness, components or sub-systems. [It] must be new to your enterprise, but it does not need to be new to the sector'.

[8] Although there exists an empirical literature that investigates firms' R&D investment differentiating between process and product innovation [e.g., Cohen and Klepper (1996), Huergo and Jaumandreu (2004), Parisi *et al.* (2006), Griffith *et al.* (2006), Petrin and Warzynski (2012), Harrison *et al.* (2014), Peters *et al.* (2017)], none of these papers uses data collected from a large and diverse sample of countries. These studies either use firm-level data from one single country (like Cohen and Klepper (1996) for US, Huergo and Jaumandreu (2004) for Spain, Parisi *et al.* (2006) for Italy, Petrin and Warzynski (2012) for Denmark, or Peters *et al.* (2017) for Germany), or from a small number of countries with similar levels of income (like Griffith *et al.* (2006) and Harrison *et al.* (2014) for France, Germany, Spain and UK). As a result, this literature is silent about correlations between income per capita and intensity of investment in process innovation relative to product innovation.

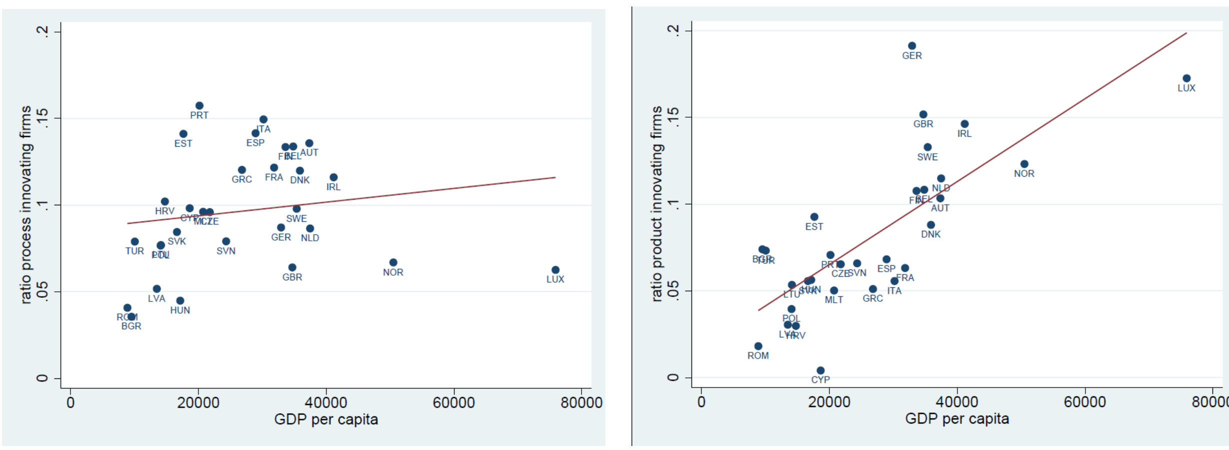
[9] There are certainly several other mechanisms that can lead to correlation patterns as those in Table 1. For

Table 1: Process Innovation and Product Innovation Intensity at Different Income Levels

	Dependent Variable			
	ratio of firms doing process and/or product innovation	ratio of firms doing process innovation	ratio of firms doing product innovation	ratio of firms doing product to firms doing process innov.
GDP per capita	0.049***	0.020*	0.040***	0.074***
	(0.012)	(0.010)	(0.009)	(0.025)
R-squared	0.37	0.12	0.42	0.24
Number countries	30	30	30	30

Robust standard errors in parentheses. All data corresponds to the *Community Innovation Survey* (2004-06). This survey was conducted on the 25 EU member states (as of 2006), plus Norway, Bulgaria, Romania, Croatia and Turkey. GDP per head data corresponds to year 2006 and is taken from Penn Tables (in PPP).
* significant 10%; ** significant 5%; *** significant 1%.

Figure 1: scatter plot of correlations in Table 1



The rest of the paper is organized as follows. Section 2 presents the setup and main assumptions of the model. Section 3 studies a simplified framework with two quality versions and where the only source of technical change is process innovations, showing that growth is eventually bound to come to a halt. Section 4 allows also the introduction of product innovations into the two-quality-version model, and shows that product innovations may help sustaining growth for longer. Section 5 extends the model to an environment with an infinite number of quality levels, which may allow positive growth in the long run. Section 6 provides some concluding remarks. All relevant proofs are relegated to Appendix A.

example, they could partly be determined by supply-side factors, such as differences in the stock of human capital or financial development, or they can be influenced by the total size of the home market via aggregate GDP. In addition, those correlations may partly reflect differences in sectoral shares between high-tech and low-tech industries, as the ratio between product and process innovations tends to be larger in former than in the latter [Peters *et al.* (2017)]. In that respect, we take the correlations in Table 1 simply as motivational evidence to our theory (and not as uniquely determined by it), while Appendix B aims at providing some complementary correlations addressing some of the above concerns.

2 Setup of the Model

Life evolves in discrete time over an infinite horizon, starting in $t = 1$. In each period t a unit mass of single-period lived households is alive. Each household i consists of two members: W_i and R_i. Household members denoted by W_i are endowed with one unit of time, which they supply inelastically to firms as labor. Those denoted by R_i are also endowed with one unit of time, but they can choose either to enjoy it as leisure time or, alternatively, to supply it as R&D effort to firms. Henceforth we will often refer to type-W individuals as *workers* and to type-R individuals as *researchers*.[10]

2.1 Technologies

The economy's output consists of a final consumption good produced by firms. The consumption good is potentially available in a discrete number of vertically ordered quality levels: $q \in Q$. We normalize the lowest value of Q to unity. We will recurrently refer to the different quality levels as *quality versions*, and to the lowest quality ($q = 1$) as the *baseline quality*.

Firms are risk neutral and are active for one period only (that is, a firm that operates in period t will close down at the end of t). There is free entry to the final good sector and opening a new firm entails no setup cost.

2.1.1 Prehistoric Technology

At the beginning of $t = 1$ the economy inherits a technology from the prehistoric period $t = 0$. The inherited technology allows transforming one unit of labor time into one unit of consumption good, but only in its baseline quality version $q = 1$. All firms active in $t = 1$ have free access to the technology inherited from $t = 0$. In addition to this, firms may hire researchers (i.e., type-R individuals) to undertake R&D effort and create blueprints for generating new technologies. In what follows, we explain in further detail the different set of technologies available in each period t and how they originate.

[10] The assumption that households always supply the unit labor time to firms is posed for analytical simplicity, as it allows us to focus on the trade-off between leisure and R&D effort. Appendix C shows how our main results hold through when we extend the model to also let households substitute labor time for leisure. In addition, that extension also relaxes the $\{0, 1\}$ discrete allocation of leisure versus effort, by allowing households to split their time endowments continuously within a unit time interval.

2.1.2 New and Inherited Technologies I: the effects of process innovation

All firms active in period t inherit at birth all the technological know-how that has been generated before t by prior events of process innovation. We broadly refer to all the technologies that were already available before period t as *inherited technologies*. Inherited technologies may be further improved upon through *current* process innovation effort.

When a generic researcher R_i alive in t is hired by a firm to exert process innovation effort he creates a (process innovation) blueprint. We denote this blueprint by $b_{i,t}^p$. Although process innovation effort always leads to a new blueprint, we assume that not all blueprints are successful in increasing labor productivity. In particular, we assume there exists a set of states of nature S, and that blueprint $b_{i,t}^p$ will lead to higher labor productivity *only* under one specific state of nature, which we denote by $s(b_{i,t}^p) \in S$. On the other hand, under any other state of nature $s \in S$ such that $s \neq s(b_{i,t}^p)$, blueprint $b_{i,t}^p$ will fail to increase labor productivity. We further assume that the set of states of nature is equal to the unit set, $S = [0, 1]$, and that $s(b_{i,t}^p) \neq s(b_{j,t}^p)$ for any $i \neq j$. The implications of this set of assumptions are twofold: *i*) for any feasible state of nature, there *always* exists one blueprint that would lead to a process innovation, *ii*) the successful (productivity–enhancing) blueprint corresponding to each feasible state of nature is *unique*.[11]

Notice that not all type-R individuals alive in t will necessarily be hired by firms to generate process innovation blueprints. It proves then convenient to define an index function $\mathbf{1}\{s(b_t^p), b_t^p = 1\}$, which equals 1 when the state of nature in t is $s(b_t^p)$ and the blueprint b_t^p is actually created in period t, and equals 0 otherwise. We can thus write down the number of process innovations generated *before* period t, denoted by A_{t-1}, as follows:

$$A_{t-1} = \begin{cases} \sum_{\tau=1}^{t-1} \mathbf{1}\{s(b_\tau^p), b_\tau^p = 1\} & \text{if} \quad t \geq 2 \\ 0 & \text{if} \quad t = 1 \end{cases} . \tag{1}$$

We can now describe formally the set of technologies available to firms active in period t resulting from process innovation effort.

[11] While these restrictions on the set S are quite artificial, they are not strictly needed for the main results of the paper. The purpose of these assumptions is to prevent the model from giving rise to equilibrium growth paths exhibiting stochastic growth rates. More precisely, if the mass of S were larger than the (unit) mass of researchers, then some periods may end up experiencing no process innovations even when all type-R individuals exert R&D effort. On the other hand, if $S \subset [0, 1]$, for a given amount of process innovation effort, some states of nature may lead to a higher number of innovations than others.

Assumption 1 *Consider some generic period $t \geq 1$. The technological options resulting from process innovation effort that are available to firms active in period t are the following ones:*

(i) Inherited technology: *Any firm active in t will be able to produce*

$$1 + \sigma A_{t-1}$$

units of the baseline quality, $q = 1$, with each unit of labor it hires, where $\sigma > 0$ and A_{t-1} is given by (1)*.*

(ii) New technologies: *If a firm active in t hires a researcher that generates the process innovation blueprint b_t^p, and the state of nature in period t turns out to be $s(b_t^p)$, then the firm will be able to produce*

$$1 + \sigma A_{t-1} + \sigma$$

units of the baseline quality, $q = 1$, with each unit of labor it hires.

The first part of Assumption 1 stipulates that the effects of process innovation on labor productivity accumulate over time, and are (freely) transferred to the next generations as inherited technologies. The second part describes the effects of current process innovation: the successful blueprint boosts labor productivity by $\sigma > 0$ units relative to the inherited technology.

2.1.3 New and Inherited Technologies II: the effects of product innovation

The economy may also have available improved technologies that originate from product innovation effort. Product innovation effort leaves labor productivity unchanged, but it allows the production of higher quality versions of the consumption good.

When a generic researcher R_i alive in t is hired by a firm to undertake product innovation effort, he will create a blueprint $b_{i,t}^q$ that may allow the production of a higher quality version of the consumption good. The blueprint $b_{i,t}^q$ will lead to a quality-upgrading innovation under the state of nature $s(b_{i,t}^q) \in S = [0, 1]$. On the other hand, under any other state of nature, $b_{i,t}^q$ will fail to generate a product innovation. Analogously to the case with process innovation, these assumptions imply that for any feasible state of nature there always exists one blueprint that would lead to a product innovation, and that the successful (quality–enhancing) blueprint in each feasible state of nature is unique.

Assumption 2 below summarizes the quality upgrading effect of product innovation effort. It also states the (physical) productivity of labor at the different levels of quality in which the final good is available. To do this, we label by q_{t-1} the *highest* quality version that is available at the end of $t-1$ (and at the beginning of t). It proves convenient again to denote by $\mathbf{1}\{s(b_t^q), b_t^q = 1\}$ an index function that is equal to 1 when the state of nature in t is $s(b_t^q)$ and the (product innovation) blueprint b_t^q is actually created in period t.

Assumption 2 *Consider some generic period $t \geq 1$, and suppose q_{t-1} is highest quality version that is available right at the beginning of t. Then:*

(i) Inherited technology: *Any firm active in t will be able to produce*

$$\frac{1+\sigma A_{t-1}}{q_{t-1}} \tag{2}$$

units of the consumption good in the quality level q_{t-1}, where

$$q_{t-1} = \begin{cases} 1+\rho \sum_{\tau=1}^{t-1} \mathbf{1}\{s(b_\tau^q), b_\tau^q = 1\} & \text{if} \quad t \geq 2 \\ 1 & \text{if} \quad t = 1 \end{cases},$$

with $\rho > 0$.

(ii) New technologies: *If a firm active in t hires a researcher that generates the product innovation blueprint b_t^q, and the state of nature in period t turns out to be $s(b_t^q)$, the firm will be able to produce*

$$\frac{1+\sigma A_{t-1}}{q_t} \tag{3}$$

units of the consumption good in the quality level q_t, where $q_t = q_{t-1} + \rho$.

Assumption 2 describes how a successful product innovation blueprint in t allows the production of a version of the good whose quality level is $\rho > 0$ units higher than the one that was available at the end of $t-1$. It also states how past product innovations alter future production possibilities: the quality version q_t will also be readily available from $t+1$ onwards as an inherited technology, without the need of any further product innovation effort.

Two additional remarks are worth stressing here. Firstly, Assumption 1 and 2 taken together imply that past process innovations generate productivity improvements that are *not* quality-specific. More precisely, the numerators in (2) and (3) entail that improvements in labor productivity owing to *prior* process innovations apply identically to *all* the existing quality versions of the consumption good.[12] Secondly, the denominators in (2) and (3) entail that the unit labor requirements are greater for higher quality versions of the consumption good.

[12] While the assumption that prior process innovations apply *identically* to all quality versions of the good may seem extreme, all our main results would still hold when process innovations spillovers applied only *partially* to higher-quality versions of the good, similarly to Jovanovic and Nyarko (1996) or Redding (2002). Essentially, what our model needs is that the positive income effects resulting from (past) process innovations do not totally vanish when introducing a product innovation.

2.2 Preferences

The utility function of a household alive in period t is given by:

$$U_t = \ln\left[\sum\nolimits_{q\in Q_t}[q\,x(q)]^q\right] + \eta(1-\varepsilon). \tag{4}$$

In (4), $x(q)$ denotes the quantity of the quality version $q \in Q_t$ consumed by the household, and $Q_t \subseteq Q$ is the subset of quality versions available in period t. Next, ε is an index function that takes the value of 1 if the type-R member of the household decided to sell his time endowment as R&D effort to a firm (either in the form of process or product innovation), and 0 if he instead chose to use it as leisure, with $\eta > 0$ being the utility of leisure.

Two important properties of (4) are worth stressing. Firstly, since the lowest value of the set Q_t is $q = 1$, the term $\sum_{q\in Q_t}[q\,x(q)]^q$ turns out to be a sum of convex functions in $x(q)$. As a consequence of this, in the optimum, households will select a *corner* solution for their consumption plan; that is, a solution characterized by $x(q) > 0$ for some $q \in Q_t$ and zero for all other quality versions. Secondly, the expression $[q\,x(q)]^q$ in (4) means that higher quality versions magnify the level of utility obtained from a given physical amount of the consumption good. Moreover, this magnifying effect becomes stronger the larger the value of physical consumption $x(q)$. This is a crucial feature of our model, as it will lead to a non-homothetic behavior in the demand for quality. In particular, the exponential effect of q on physical consumption $x(q)$ leads to demand functions where the willingness to pay for higher versions of the final good is increasing in the level of spending of the household.[13]

2.3 Timing of Events

In each period t, actions take place with the following within-period timing:

1. A unit mass of households comprising one type-W and one type-R individual come to life. All firms active in t inherit all the technologies that were available at the end of $t-1$.

2. Type-W individuals (i.e., workers) sell their labor endowments to firms in exchange of a wage. Type-R individuals (i.e., researchers) decide whether to use their time endowment

[13] In growth models with homothetic preferences, the distinction between reductions in costs per physical unit of production (i.e., process innovation) and increases in quality per physical unit (i.e., product innovation) may usually be blurred, since those sources of technical change can often be re-labeled to become isomorphic. Our nonhomothetic preference structure in (4) actually turns the distinction between quality improvements and physical productivity improvements economically meaningful, as the income-dependent willingness to pay for quality upgrading implies that those sources of growth cannot be taken simply as isomorphic to one another.

(ε_i) as leisure or to sell it as R&D effort to a firm in exchange of a salary. In case of the latter, the firm decides whether to request process innovation or product innovation effort from the researcher.[14] (For the entire paper, we use *wage* to refer to the pay received by workers, and *salary* for the pay received by researchers.)

3. All new blueprints are created.

4. Nature reveals the state of nature in period t.

5. Production and consumption take place.

The timing of the events stipulates that, when researchers sell their R&D effort to firms, they do so *before* the state of nature of period t becomes publicly known. This in turn means that firms will sometimes pay researchers a salary to create blueprints that (ex-post) end up being useless. Given the preference structure of households in (4), this timing may however be seen as optimal from the point of view of firms and households. Firms are risk neutral, while households are risk averse. As a result, since the actual return of exerting R&D effort is (ex-ante) unknown, firms will end up absorbing all the risk involved in innovation activities by paying researchers a fixed salary before the uncertainty is revealed.

3 Endogenous Growth via Process Innovation

In this section, we study the dynamic behavior of the economy under the assumption that only process innovations are feasible. Doing this allows a cleaner description of the conditions under which process innovation arises in equilibrium, and how it endogenously generates its own tendency to eventually come to a halt.

For the rest of the paper, we take the baseline quality version of the consumption good as the *numeraire*. Hence all prices will be expressed as relative to the price of $q = 1$. In addition, for most part of the paper, we further simplify the analysis by considering an environment where there exist only two potential quality versions of the consumption good; that is, $Q=\{1, 1+\rho\}$. (In Section 5 we show how the model easily extends to a more general environment with an infinite number of quality versions, and the dynamic implications of this.) Finally, throughout the paper, we restrict all our analysis to symmetric equilibria under pure strategies.[15]

[14] None of the results of the model would change if we instead assumed that it is the researcher who chooses which type of R&D effort to sell to firms, as long as firms can then choose to pay different salaries according to the type of innovation effort they buy.

[15] There exist some parametric configurations of the model under which mixed-strategy equilibria would arise

3.1 Process Innovation Effort in t=1

Recall from Assumption 1 that in $t = 1$ any firm can produce one unit of $q = 1$ with one unit of labor time. As a result, in equilibrium, firms will have to pay a wage equal to one for each unit of labor they hire from households. We label this wage by $w_1 = 1$.

Households may also receive earnings from selling the time endowment of their type-R member as R&D effort to firms. Free entry and firm competition imply that, in equilibrium, researchers must be paid a salary equal the *expected* return associated to the blueprints they create. To compute this value, consider a hypothetical firm that hires a researcher who creates the process innovation blueprint b_1^p, and suppose $s(b_1^p) \in S$ turns out to be the state of nature that (ex post) holds in $t = 1$. Then, this firm will be able to produce $1 + \sigma$ units of the baseline quality version of the final good by hiring all the available labor supply in t. Since each unit of labor costs $w_1 = 1$, the blueprint b_t^p will thus end up generating a return to the firm equal to σ. Finally, give that all blueprints are (ex ante) identical in terms of their success probability, in equilibrium, all researchers should be paid a salary equal to σ for their process innovation effort (which is equal the *expected* return that their process innovation effort generates).[16]

Let $Y_{i,1}$ denote the lifetime earnings of a generic household i at the end of $t = 1$. We can split $Y_{i,1}$ in two different components: i) the earnings generated by W_i (the worker) which is equal to the wage, $w_1 = 1$; ii) the earnings generated by R_i (the researcher), in case R_i decided to sell his time endowment to a firm in exchange of a salary equal to σ. Thus, when R_i sells his time endowment as R&D effort to a firm,

$$Y_{i,1}\left(\varepsilon_{i,1} = 1\right) = 1 + \sigma. \tag{5}$$

Instead, if R_i chooses to use his time endowment as leisure,

$$Y_{i,1}\left(\varepsilon_{i,1} = 0\right) = 1. \tag{6}$$

Using (4), it follows that a generic household i alive in $t = 1$ will set $\varepsilon_{i,1} = 1$ if and only if: $\ln(Y_{i,1}\left(\varepsilon_{i,1} = 1\right)) > \ln(Y_{i,1}\left(\varepsilon_{i,1} = 0\right)) + \eta$. (For the rest of the paper we assume that in case of

alongside symmetric pure-strategy equilibria. Nevertheless, when mixed-strategy equilibria do exist, they turn out to be locally unstable (see footnote 20 in Section 4.2 for some further discussion on this issue).

[16] Rigorously speaking, since we have a continuum of households i such that $\int_0^1 di = 1$, a blueprint created by R_i carries a probability di to become the successful blueprint, generating thus an expected return $\left[\int_0^1 (1+\sigma)dj - \int_0^1 1dj\right] di = \sigma di$ when the firm that purchased it hires also the *entire* available labor supply. From now on, to keep a lighter notation, we simply write σ as the expected return of any process innovation blueprint. Such abuse of notation, in turn, entails that we think of the utility of leisure as having a comparable measure to household income (i.e., we take the utility of leisure of household i as being equal to $\eta\, di$).

indifference $\varepsilon_i = 0$ is chosen.) Hence, using (5) and (6), and noting that $Y_{i,1}$ depends only on the action by household i (i.e., $\varepsilon_{i,1}$), we can finally obtain the condition that ensures that, in equilibrium, households alive in $t = 1$ will exert process innovation effort. Namely:

$$\ln(1+\sigma) > \eta. \tag{7}$$

Otherwise, they will all set $\varepsilon_1 = 0$.

Condition (7) shows (quite intuitively) that a larger value of σ is instrumental to sustaining an equilibrium with positive process innovation effort. An equilibrium where $\varepsilon_1 = 1$ requires that the *additional utility* obtained from $Y_{i,1}(\varepsilon_{i,1} = 1)$, relative to that obtained from $Y_{i,1}(\varepsilon_{i,1} = 0)$, more than compensates the disutility of effort incurred when $\varepsilon_{i,1} = 1$, which leads (7).

3.2 Process Innovation Effort in a Generic t

Assumption 1 states that any firm active in period t can freely use the inherited technology from $t-1$, and produce $1 + \sigma A_{t-1}$ units of baseline quality with each unit of labor it hires. Competition among firms for workers will thus lead to an equilibrium wage:

$$w_t = 1 + \sigma A_{t-1}. \tag{8}$$

Any new process innovation blueprint created in period t will boost labor productivity, in expectation, by σ units of the baseline quality good. Therefore, in equilibrium, the salary that firms would need to pay to hire a researcher will always be equal to σ.

Using this result, together with (8), we can now generalize (5) and (6) for an economy whose inherited technology embodies A_{t-1} past process innovations. In this case, the lifetime earnings of a household in t as a function of ε_t will be given by:

$$Y_t(\varepsilon_t = 1) = 1 + \sigma A_{t-1} + \sigma, \tag{9}$$

$$Y_t(\varepsilon_t = 0) = 1 + \sigma A_{t-1}. \tag{10}$$

From (9) and (10), it follows that a necessary and sufficient condition for an equilibrium with $\varepsilon_t^* = 1$ to hold, is that $\ln(1 + \sigma A_{t-1} + \sigma) > \ln(1 + \sigma A_{t-1}) + \eta$. This condition leads to the following result.

Lemma 1 *Consider an economy in t which up until $t-1$ has gone through A_{t-1} periods where, in equilibrium, all type-R household members have been hired to undertake process innovation effort. Then, the equilibrium in period t will feature $\varepsilon_t^* = 1$ if and only if:*

$$\ln\left(1 + \frac{\sigma}{1+\sigma A_{t-1}}\right) > \eta. \tag{11}$$

The result in Lemma 1 generalizes condition (7) to any period t. This simple generalization yields also some additional insights. First, notice that $\sigma/(1+\sigma A_{t-1})$ is strictly decreasing in A_{t-1}, and it converges to zero when $A_{t-1}\to\infty$. Hence, there will always be a value of A_{t-1} large enough such that (11) will fail to hold, and period t will thus feature an equilibrium with $\varepsilon_t^*=0$ (that is, where the economy experiences no process innovation effort). Moreover, the ratio $\sigma/(1+\sigma A_{t-1})$ is strictly increasing in σ. As a result, the threshold value of A_{t-1} beyond which process innovation will stop tends to be greater for economies with a larger σ. The exact value of this threshold is pinned down by the level of A_{t-1} that makes left-hand side of (11) equal to η, namely:

$$\overline{A}(\sigma)\equiv\frac{1}{(e^{\eta}-1)}-\frac{1}{\sigma}. \tag{12}$$

Using (12), we can now observe that Lemma 1 implies that there is a maximum number of (subsequent) periods such that the economy will be able to exhibit an equilibrium with positive process innovation effort. Furthermore, the fact that $\overline{A}'(\sigma)>0$ also means that economies with a larger σ are able to sustain an equilibrium with positive process innovation effort for longer.[17]

To conclude this section, we can finally describe the full dynamic behavior of A_t in this version of the model where only process innovation effort is allowed. In order to deal with integer issues, we introduce one additional threshold:

Definition 1 *Let $\overline{t}(\sigma)\equiv\text{integer}\{\overline{A}(\sigma)+1\}$, where $\overline{A}(\sigma)$ is defined by (12).*

Proposition 1 *When condition (7) holds, implying that $\overline{A}(\sigma)>0$, the economy will experience an equilibrium with positive process innovation effort in all periods from $t=1$ until $t=\overline{t}(\sigma)\geq 1$. In any period $t>\overline{t}(\sigma)$, the economy will experience an equilibrium without process innovation effort. This, in turn, implies that $A_t=t$ whenever $t\leq\overline{t}(\sigma)$, while $A_t=\overline{t}(\sigma)$ for all $t>\overline{t}(\sigma)$.*

Proposition 1 shows that, when condition (7) holds, the economy will keep investing in process innovation in each period $t\geq 1$ until reaching $t=\overline{t}$ (notice, however, that it may well be the case that $\overline{t}=1$).[18] While this happens productivity will accordingly grow, which is reflected in the fact that $A_t=t$ whenever $t\leq\overline{t}$. Once the economy goes past period $\overline{t}$, process innovation effort stops forever, and A_t remains thereafter constant and equal to $\overline{t}$.

The intuition behind the fact that process innovation eventually comes to halt hinges on the decreasing marginal utility of consumption. When generic household i is contemplating

[17] Notice also that when $\sigma\leq e^{\eta}-1$, we have that $\overline{A}(\sigma)\leq 0$, and no process innovation will ever take place. In fact, $\sigma>e^{\eta}-1$ is just the same condition as (7) for $t=1$.

[18] From now on, to lighten up notation, we will often write down the period-threshold function $\overline{t}(\sigma)$ simply as $\overline{t}$, and avoiding thus making its dependence on σ explicit when this creates no confusion.

whether or not to sell the time endowment of R_i as R&D effort to a firm, the household faces a trade-off between higher consumption versus higher leisure. Setting $\varepsilon_t = 1$ allows a level of consumption equal to $Y_t(\varepsilon_t = 1) = 1+\sigma A_{t-1}+\sigma$. On the other hand, $\varepsilon_t = 0$ leads a lower level of consumption, $Y_t(\varepsilon_t = 0) = 1 + \sigma A_{t-1}$, but it yields an additional non-pecuniary benefit, η. The difference between $Y_t(\varepsilon_t = 1)$ and $Y_t(\varepsilon_t = 0)$ equals always σ. However, as the economy goes through subsequent rounds of process innovation and A_{t-1} accordingly rises, both $Y_t(\varepsilon_t = 1)$ and $Y_t(\varepsilon_t = 0)$ increase. In a context with decreasing marginal utility of consumption, higher values of $Y_t(\varepsilon_t = 1)$ and $Y_t(\varepsilon_t = 0)$ in turn imply that the consumption gap between them tends to become less and less appealing relative to the required leisure sacrifice η.

4 Introducing Product Innovation

We proceed now to introduce product innovation effort into the model. We consider the situation where the only quality version that was available at the end of $t-1$ was the baseline quality; namely, $q_{t-1} = 1$. In this context, product innovation effort in t he will lead to blueprints that may allow the production of the quality version $q_t = 1 + \rho$.

Before moving on to the full equilibrium analysis of the model, it proves convenient to first address the following two questions: *i*) what is the expected return that a product innovation blueprint yields?; *ii*) what is the price that a firm producing the quality version $q_t = 1 + \rho$ will charge for this commodity?

Lemma 2 *Consider a generic researcher R_i alive in period t who is hired by a firm to create a product innovation blueprint. The expected return of the product innovation blueprint designed by R_i is given by:*

$$\pi_{i,t}^{q} = P_t \frac{1 + \sigma A_{t-1}}{1 + \rho} - (1 + \sigma A_{t-1}), \tag{13}$$

where P_t in (13) denotes the market price of the quality version $q_t = 1+\rho$, which in equilibrium will be given by:

$$P_t = (1 + \rho)\, Y_t^{\rho/(1+\rho)}. \tag{14}$$

The first result in Lemma 2 shows the expected return (net of workers wage cost) generated by a blueprint that may allow producing the higher-quality version ($q_t = 1 + \rho$), given its price P_t. Notice that competition by firms for researchers' effort time entails that, in an equilibrium where firms hire researchers to exert product innovation effort, the salary offered to them will have to be equal to the RHS of (13) with $P_t = (1 + \rho)\, Y_t^{\rho/(1+\rho)}$.

The second result in Lemma 2 shows in (14) that the price charged for the version of the good of quality $1+\rho$ rises with the lifetime earnings of households alive in t. The responsiveness of P_t to Y_t is a direct implication of our preference structure in (4), where the quality index q magnifies the utility derived from the physical quantity of consumption, $x(q)$. Such preference structure leads to a nonhomothetic behavior where the willingness to pay for the version of the good with $q = 1 + \rho$ rises with Y_t.[19]

The equilibrium value of P_t thus depends on the level of Y_t, which is itself also an equilibrium object. In particular, the level of Y_t will be ultimately a function of the innovation effort equilibrium choices in t. Equation (9) stated how Y_t rises with subsequent rounds of process innovation effort. The following lemma complements those results when we allow for product innovation effort as well.

Lemma 3 *Consider a generic economy in period t. Suppose that up until period $t-1$ there were A_{t-1} periods in which all type-R household members were hired to exert process innovation effort. In addition, suppose that $q_{t-1} = 1$ (i.e., by the end of period $t-1$ only the baseline quality was available). Then, if in period t all type-R household members are hired to exert product innovation effort, we will have that:*

$$Y_t = (1 + \sigma A_{t-1})^{1+\rho}. \tag{15}$$

Lemma 3 shows that the larger the value of A_{t-1} the greater Y_t will be in an equilibrium with product innovation effort. In addition, it is interesting to notice that when $A_{t-1} = 0$, the expression (15) yields $Y_t = 1$. In other words, in the absence of *any* previous rounds of process innovations, product innovation effort cannot induce by itself a rise in incomes. Therefore, some initial rounds of process innovation effort are required in order to ignite income growth. In turn, as consumers' incomes grow, this may endogenously generate a change in relative profits, and shift R&D effort from process innovation to product innovation. We next study how this particular growth sequence may arise as an equilibrium outcome of the model.

Before moving on, we need some additional notation to distinguish the type of innovation effort carried our. We use henceforth $\varepsilon_{t,p} = 1$ for process innovation effort, and $\varepsilon_{t,q} = 1$ for product innovation effort in t. We continue to denote by $\varepsilon_t = 0$ the choice of consuming leisure.

[19] The equilibrium value of P_t in (14) is that one that leaves households indifferent between buying the higher-quality version at the price P_t and buying the baseline quality version (which being the *numeraire* carries a price equal to one). Given our nonhomothetic structure, this indifference price rises with the level of income Y_t.

4.1 (Absence of) Product Innovation Effort in t=1

We show first that an equilibrium with product innovation cannot arise in $t = 1$. The next subsection in turn proceeds to study whether an equilibrium with product innovation effort may eventually arise at some point along the growth path.

Notice that, in an equilibrium where $\varepsilon^*_{1,q} = 1$ (i.e., where all types R_i alive in $t = 1$ are hired to exert product innovation effort), households' lifetime earnings in $t = 1$ would follow from (15) with $A_{t-1} = 0$. This would yield $Y_1 = 1$, and using Lemma 2 would in turn imply that $\pi^q_{i,t} = 0$, and hence firms would *not* be willing to pay a positive salary in exchange of product innovation effort. Given that offering product innovation effort to firms entails giving up the utility of leisure η, no household would thus find it optimal to do so in this case.

The above result shows that an equilibrium where $\varepsilon^*_{1,q} = 1$ cannot exist. Once we allow for product innovation effort, pinning down the type of equilibrium that arises in $t = 1$ requires also analyzing possible individual deviations to $\varepsilon_{i,1,q} = 1$ from situations where either $\varepsilon^*_{1,p} = 1$ or $\varepsilon^*_1 = 0$. Consider, firstly, the case in which all type-R households members are hired to exert process innovation effort in equilibrium (i.e., $\varepsilon^*_{1,p} = 1$). The expected return of shifting one generic researcher i to exerting product innovation effort would be equal to $(1 + \sigma)^{\rho/(1+\rho)} - 1$.[20] This value is strictly smaller than σ, which is the expected return of process innovation effort. Next, we can also observe that no firm could possibly induce a deviation to $\varepsilon_{i,1,q} = 1$ by some generic R_i from an equilibrium with $\varepsilon^*_1 = 0$, while simultaneously satisfy the zero-profit condition. This result follows from a similar reasoning as the one precluding the existence of an equilibrium with $\varepsilon^*_{1,q} = 1$: when households set $\varepsilon^*_1 = 0$, product innovation blueprints generate a expected return equal to zero, hence no firm would be willing to pay a positive salary for product innovation effort by researchers.

4.2 Product Innovation Effort in a Generic t

For an equilibrium with product innovation effort to arise in t two non-deviation conditions must be satisfied when $\varepsilon^*_{t,q} = 1$ holds. The first is that no single household i must prefer to deviate consuming the time endowment of R_i as leisure. The second is that no firm must prefer to request a researcher to exert process innovation effort instead of product innovation effort.

The previous subsection shows that the first condition above fails to be satisfied during $t = 1$. The reason for this is that, in the absence of any previous process innovation effort (that

[20] To obtain this using the expression in (13), note that the price that firms would be able to charge for the higher-quality version when $\varepsilon^*_{1,p} = 1$ is $P_1 = (1 + \rho)(1 + \sigma)^{\rho/(1+\rho)}$, since in this case we have that $Y_1 = 1 + \sigma$.

is, when $A_{t-1} = 0$), equation (15) yields $Y_t = 1$, which (given our nonhomothetic preference structure) turns out to be too small to make product innovation effort profitable. Notice, however, that according to (15) the level of Y_t in an equilibrium with product innovation effort is increasing in A_{t-1}. This suggests that there may exist a value of A_{t-1} large enough to be able to support such an equilibrium. The next lemma lays out this result formally.

Lemma 4 *Consider an economy in period t that has previously undergone A_{t-1} periods in which all type-R household members had been hired to exert process innovation effort. Suppose all type-R household members alive in t are hired to exert product innovation effort.*

i) If

$$\rho \ln (1 + \sigma A_{t-1}) > \eta, \tag{16}$$

no single household i alive in t will prefer consuming the time endowment of R_i as leisure.

ii) If

$$(1 + \sigma A_{t-1})^{1+\rho} > 1 + \sigma A_{t-1} + \sigma, \tag{17}$$

no single firm active in t will request a researcher to exert process innovation effort instead of product innovation effort.

Lemma 4 presents the two non-deviation conditions that an equilibrium with product innovation must satisfy in period t. These conditions will only hold when A_{t-1} is sufficiently large. In other words, unless the economy has previously undergone a sufficiently large number of rounds of process innovations, it will *not* be able to sustain an equilibrium with product innovation effort in t. While conditions (16) and (17) ensure the existence of an equilibrium with product innovation in t, they do not rule out the possibility that other types of symmetric equilibria in pure strategies may exist as well. The next proposition provides a more general description of the equilibrium that arises in a generic period t.

Proposition 2 *Consider an economy in period t that has previously undergone A_{t-1} periods with process innovations, and for which both (16) and (17) hold true. Then:*

i) There exists an equilibrium in period t in which all type-R household members exert product innovation effort (i.e., $\varepsilon^*_{t,q} = 1$).

ii) The equilibrium with $\varepsilon^*_{t,q} = 1$ is the unique pure-strategy symmetric equilibrium in period t, unless the following two conditions are also verified:

$$\ln \left(1 + \frac{\sigma}{1 + \sigma A_{t-1}}\right) \leq \eta \tag{18}$$

$$\frac{\rho}{1+\rho} \ln(1 + \sigma A_{t-1}) \leq \eta. \tag{19}$$

In particular, when (18) and (19) hold true, alongside with $\varepsilon^*_{t,q} = 1$, there also exists an equilibrium in which all R_i alive in t consume time endowment as leisure (i.e., $\varepsilon^*_t = 0$).

Proposition 2 shows that when the two non-deviation conditions stipulated in Lemma 4 are satisfied, the economy will exhibit an equilibrium in period t with product innovation effort. In addition, this equilibrium is also unique (within the class of pure-strategy symmetric equilibria), unless the parametric configuration of the model is such that both (18) and (19) hold (together with the conditions in Lemma 4).

The multiple equilibria case described in Proposition 2 arises because of the possibility of coordination failures under some parametric configurations. In this paper, we are mostly interested in studying under which conditions will economies be able to support long-run growth, rather than in the possibility of coordination failures preventing (potential) growth from materializing. For this reason, in the next subsection we will disregard the equilibrium with $\varepsilon^*_t = 0$, when it arises as a coordination failure in a context of multiple equilibria. Nevertheless, in Section 4.5 we will return to the issue of multiple equilibria, and provide an intuition for the mechanism leading to coordination failures.[21]

4.3 From Process Innovation to Product Innovation

Although Proposition 2 specifies the conditions that would lead to the equilibrium with product innovation effort, it still leaves one crucial question pending: whether or not an economy will actually be able to (endogenously) generate a level of A_{t-1} large enough to make (16) and (17) hold simultaneously. In fact, if it fails to do so (because the incentives to keep undertaking further process innovation wane too quickly), the equilibrium with product innovation effort will never materialize. We proceed to study now the conditions required for a successful transition from a equilibrium with process innovation to one with product innovation.

Some additional notation will prove useful for future reference. Firstly, we will denote by $\underline{A}_1$ the value of A_{t-1} that makes the LHS of (16) be equal to η. Namely:

$$\underline{A}_1 \equiv \frac{e^{\eta/\rho} - 1}{\sigma}. \tag{20}$$

Secondly, we will denote by $\underline{A}_2$ the value of A_{t-1} that equals the LHS of (17) to its RHS. In

[21] When the model yields both $\varepsilon^*_{t,q} = 1$ and $\varepsilon^*_t = 0$ as Nash equilibria, a third type of equilibrium will also exist involving mixed-strategies among product innovation effort and leisure. This mixed-strategy equilibrium is, however, locally unstable.

this case, there is no general explicit solution for $\underline{A}_2$, which is thus defined implicitly by:

$$\frac{(1+\sigma\underline{A}_2)^{1+\rho}}{1+\sigma\underline{A}_2+\sigma} \equiv 1. \tag{21}$$

Both $\underline{A}_1$ and $\underline{A}_2$ are strictly decreasing in σ.[22] When $A_{t-1} > \underline{A}_1$ and $A_{t-1} > \underline{A}_2$, there is an equilibrium with positive product innovation effort in t. Since both conditions must hold for this, we can simply combine $\underline{A}_1$ and $\underline{A}_2$ together by defining:

$$\underline{A}(\sigma) \equiv \max\{\underline{A}_1, \underline{A}_2\}, \tag{22}$$

where in (22) we make it explicit the dependence of $\underline{A}$ on σ. The threshold $\underline{A}(\sigma)$ essentially pins down the *minimum* value that A_{t-1} must reach for the economy to be able to switch to an equilibrium with product innovation effort in period t.

Since the model takes place in discrete time, in order to deal with integer issues, we must also introduce a *period* threshold.

Definition 2 *Let* $\underline{t}(\sigma) \equiv \text{integer}\{\underline{A}(\sigma)+1\}$, *where* $\underline{A}(\sigma)$ *is defined by* (22).

The value of $\underline{t}(\sigma)$ pins down the *minimum* number of periods the economy must sustain an equilibrium with process innovation effort before it can switch to an equilibrium with product innovation effort.

Proposition 3 *Consider an economy that satisfies condition* (7). *Depending on the specific values taken by* $\bar{t}(\sigma)$ *and* $\underline{t}(\sigma)$, *this economy may or may never be able to switch at some point to an equilibrium with product innovation effort. In particular:*

i) If $\bar{t}(\sigma) < \underline{t}(\sigma)$, *the economy will experience process innovation effort until period* $t = \bar{t}$. *From* $t > \bar{t}$ *onwards the economy will stop carrying out any type of innovation effort.*

ii) If $\bar{t}(\sigma) \geq \underline{t}(\sigma)$, *the economy will experience process innovation effort until period* $t = \underline{t}$, *and in* $t = \underline{t}+1$ *the economy will be able to switch to an equilibrium with product innovation effort.*

Proposition 3 shows first that when parametric conditions lead to $\bar{t}(\sigma) < \underline{t}(\sigma)$, the economy will *never* manage to switch to an equilibrium with product innovation. Importantly, in these cases, process innovation and income growth will eventually come to a halt. This will happen in $t = \bar{t}$. From then on, the value of A_t will remain constant at $\bar{t}$, implying in turn that incomes will also stay fixed thereafter, at the level $Y_t = 1 + \sigma\bar{t}$ for all $t \geq \bar{t}$.

[22] The fact that $\underline{A}_1$ decreases with σ can be observed directly from the expression in (20); a formal proof that $\underline{A}_2$ decreases with σ can be found in Appendix D.

Conversely, when $\bar{t}(\sigma) \geq \underline{t}(\sigma)$, an equilibrium with product innovation effort will arise in $t = \underline{t} + 1$. The economy will experience an initial phase of growth driven by process innovation effort until $t = \underline{t}$. As incomes rise during this phase, the (implicit) willingness to pay for the higher-quality version increases. Eventually, at $t = \underline{t} + 1$ the willingness to pay for $q = 1 + \rho$ becomes high enough to turn product innovation effort more profitable than process innovation effort. At this point, the growth-regime switch can take place.

An important question to address is what turns the condition $\bar{t}(\sigma) \leq \underline{t}(\sigma)$ more likely to hold. Recall that $\overline{A}(\sigma)$ is increasing in σ, while $\underline{A}(\sigma)$ is decreasing in σ. In addition, from the expressions in (20) and (21), it follows that the threshold $\underline{A}(\sigma)$ also satisfies $\lim_{\sigma \to 0} \underline{A}(\sigma) = \infty$ and $\lim_{\sigma \to \infty} \underline{A}(\sigma) = 0$, while from (12) we can observe that $\lim_{\sigma \to \infty} \overline{A}(\sigma) = (e^{\eta} - 1)^{-1} > 0$. As a result, given the way $\bar{t}(\sigma)$ and $\underline{t}(\sigma)$ are defined, there must always exist a value of σ high enough to ensure that condition $\bar{t}(\sigma) \leq \underline{t}(\sigma)$ holds true. We state this result more formally in the following corollary.

Corollary 1 *There exists a strictly positive and finite cut-off value, $\widehat{\sigma} > e^{\eta} - 1$, such that when $\sigma > \widehat{\sigma}$, the condition $\bar{t}(\sigma) \leq \underline{t}(\sigma)$ holds true.*

4.4 From Product Innovation (Back) to Process Innovation

In our model, process innovation effort exhibits an inherent tendency to come to a halt. For this reason, it becomes crucial that the switch to an equilibrium with product innovation effort takes place soon enough; otherwise the switch will simply end up not happening at all. Naturally, product innovation effort helps sustaining positive growth while it takes place. However, there is one additional positive effect that product innovation exerts on growth: product innovations may also boost the incentives to further undertake process innovation effort in future periods.

The underlying reason why the incentives to undertake process innovation effort exhibit an intrinsic decaying tendency rests on the decreasing marginal utility of consumption. As process innovations lead to an expansion of the physical production of the baseline quality ($q = 1$), the additional utility obtained from higher consumption declines, hurting in turn the profit derived from further rounds of process innovation. Product innovation effort works, however, on a rather distinct dimension: it leads to higher utility by each unit of physical consumption. Furthermore, the marginal utility of consumption declines more slowly for higher quality versions than for lower quality ones. Quality upgrading thus relaxes the depressing effect that decreasing marginal utility of consumption imposes on the incentives to further raise physical production via process innovation.

In the sake of brevity, in this subsection we focus only on the case in which $\overline{t}(\sigma) \geq \underline{t}(\sigma)$ holds. This means that the growth path of the economy is driven by positive process innovation effort until period $t = \underline{t}$, with $A_t = t$ during that phase. Next, at $t = \underline{t} + 1$, the economy is able to switch to an equilibrium with product innovation effort. This will allow the production of the quality version $q = 1 + \rho$ in $t = \underline{t} + 1$, and also in all $t > \underline{t} + 1$.

Consider now an economy in $t = \underline{t} + 2$, right after an equilibrium with product innovation effort took place. Firms active in this period will inherit a technology that allows them to produce $(1 + \sigma\underline{t})/(1 + \rho)$ units the quality version $q = 1 + \rho$ with one unit of labor. The question to address now is whether the economy will manage to exhibit an equilibrium in $t = \underline{t} + 2$ where households will offer R's time endowment to firms as process innovation effort, or if they will prefer to consume it as leisure.

When a generic R_i alive in $t = \underline{t} + 2$ undertakes process innovation effort, this will yield a blueprint whose expected return is given by:

$$\pi^p_{i,\underline{t}+2} = P_{\underline{t}+2} \frac{1 + \sigma(1 + \underline{t})}{1 + \rho} - w_{\underline{t}+2},$$

where,

$$P_{\underline{t}+2} = (1 + \rho) Y_{\underline{t}+2}^{\rho/(1+\rho)} \quad \text{and} \quad w_{\underline{t}+2} = P_{\underline{t}+2} \frac{1 + \sigma\underline{t}}{1 + \rho}.$$

Again, competition by firms implies that in equilibrium, $\pi^p_{i,\underline{t}+2}$ will be equal to the salary offered to R_i. Hence, when household i sets $\varepsilon_{i,p} = 1$,

$$Y_{i,\underline{t}+2}(\varepsilon_{i,p} = 1) = Y_{\underline{t}+2}^{\rho/(1+\rho)} \left[1 + \sigma(1 + \underline{t})\right]. \tag{23}$$

On the other hand, if R_i consumes his time endowment as leisure, household i will only obtain the wage $w_{\underline{t}+2}$. Thus,

$$Y_{i,\underline{t}+2}(\varepsilon_{i,p} = 0) = Y_{\underline{t}+2}^{\rho/(1+\rho)} (1 + \sigma\underline{t}). \tag{24}$$

Comparing (23) and (24), leads finally to the following result:

Proposition 4 *Consider an economy for which the condition $\overline{t}(\sigma) \geq \underline{t}(\sigma)$ is satisfied. Following an equilibrium with product innovation effort in $t = \underline{t} + 1$, if the following condition holds:*

$$(1 + \rho)\left[\ln(1 + \sigma\underline{t} + \sigma) - \ln(1 + \sigma\underline{t})\right] > \eta, \tag{25}$$

then the economy will experience an equilibrium with process innovation effort in $t = \underline{t} + 2$. In other words, the equilibrium in $t = \underline{t} + 2$ will feature $\varepsilon^*_{\underline{t}+2,p} = 1$.

To interpret Proposition 4, it proves insightful to compare condition (25) vis-a-vis (11). Recall that along a growth path with positive process innovation effort we have $A_t = t$. Given this, according to condition (11), an economy that managed to sustain process innovation effort until $t = \underline{t}$ would next need to satisfy

$$\ln\left(1 + \sigma\underline{t} + \sigma\right) - \ln\left(1 + \sigma\underline{t}\right) > \eta, \tag{26}$$

in order to be able to keep sustaining further process innovation effort *after* period $\underline{t}$. The situation is slightly different for an economy that exhibited a growth path with process innovation effort until $\underline{t}$, and switched to an equilibrium with product innovation effort in $\underline{t} + 1$. In this case, to be able to support further process innovation effort after period $\underline{t} + 1$, the economy will need to satisfy (25). This condition is actually *weaker* than (26) since $\rho > 0$. Hence, in our model, product innovation effort may also foster growth by reinvigorating the incentives to carry out further process innovation effort in future periods.

4.5 Coordination Failures and Low-Quality Traps

Proposition 2 showed that under certain parametric configurations our model exhibits multiple equilibria. In one equilibrium, expectations coordinate on $\varepsilon^*_{t,q} = 1$. This is the equilibrium we studied in the previous two subsections, where product innovation keeps the economy on a positive growth path during t (and, possibly, it also fosters future growth by regenerating the incentives for further process innovation after t). The other equilibrium is, instead, the result of a coordination failure: agents expect that no product innovation effort to be exerted, which in turn ends up curbing incentives to exert product innovation effort.

The possibility of coordination failures rests on the combined effect of a pecuniary and a non-pecuniary positive externality associated with product innovations. The intuition behind the pecuniary externality is quite straightforward. As more R_i household members exert product innovation effort (instead of consuming leisure) the value of Y_t increases. This in turn means that a firm selling the higher-quality version of the good can charge a higher price (P_t) for it, which ultimately translates into a greater expected return from the blueprints.

The intuition for the non-pecuniary externality in more subtle. It relates to a higher probability of expansion in the set of *available* quality versions (Q_t) as more R_i agents exert product innovation effort. Recall that each product innovation blueprint will lead to a successful product innovation only in one specific state of nature. As consequence, the larger the mass of R_i who actually exert product innovation effort, the greater the probability that one of those blueprints will (ex-post) lead to successful product innovation. Households derive more utility from the higher-quality version than from the baseline quality. Hence, an increased probability that the higher-quality version will be marketed in period t will *indirectly* raise as well the incentives for R_i agents to offer their time endowment as product innovation effort to firms.[23]

5 Unbounded Quality Levels and Long-Run Growth

So far we have studied an environment with only two levels of quality; i.e., $Q = \{1, 1+\rho\}$. While this simplified framework is able to convey our main insights in terms of feedbacks between process and product innovation effort, it cannot generate dynamics with rising incomes in the long run. In particular, when Q is bounded above, innovation and growth will eventually stop, no matter the parametric configuration of the model. This section extends the previous model by allowing an infinite number of quality levels. Interestingly, we show that in this case those economies that manage to switch to an equilibrium with quality upgrading in $t = \underline{t} + 1$ (as described in Proposition 3) will turn out to be able to sustain positive growth forever.

Before moving on to show this result formally, it should be first straightforward to note that allowing an infinite number of quality levels will not alter any of our previous results for economies that *fail* to reach at some point an equilibrium with product innovation. In other words, even when Q is unbounded above, if the parametric configuration of the model is such that $\bar{t}(\sigma) < \underline{t}(\sigma)$, the economy will experience process innovation effort until $t = \bar{t}$, and it will stop carrying out any type of innovation effort after $\bar{t}$.

For the rest of this section we will then focus on the case when $\bar{t}(\sigma) \geq \underline{t}(\sigma)$. As we have already seen, this economy will feature an equilibrium with process innovation until period $t = \underline{t}$, and it will switch to an equilibrium with product innovation in $t = \underline{t}+1$. The question to address now is two-fold: whether the economy will be able to sustain an equilibrium with *some* type of innovation effort in $t = \underline{t} + 2$, and in case it is able to do so *which* type of innovation effort it will be. Naturally, if the economy experiences some type of innovation effort in $\underline{t} + 2$, the very same question will arise again in $\underline{t} + 3$, and so on and so forth.

The following lemma shows an important preliminary result regarding the income path in the long run when the set Q comprises an infinite number of quality versions.

[23] While the pecuniary externality linked to product innovation effort has a similar flavour to those present in the Big Push literature [e.g., Murphy, Shleifer and Vishny (1989)], the non-pecuniary externality is conceptually very different from those studied before in the context of coordination failures and poverty traps. This externality is caused by an expansion in the consumption space when the economy experiences a product innovation, which in turn alters agents' marginal rate of substitution between leisure and commodity consumption in favour of the latter (thereby indirectly enhancing agents' incentives to undertake product innovation effort).

Lemma 5 *When* $Q = \{1, 1+\rho, 1+2\rho, ...\}$*, economies that satisfy the condition* $\bar{t}(\sigma) \geq \underline{t}(\sigma)$ *will always be able to sustain an equilibrium with some type of innovation effort.*

Lemma 5 essentially states that economies which are able to switch to an equilibrium with product innovation in period $\underline{t}+1$, will also be able sustain an equilibrium with positive innovation effort in all periods after $\underline{t}+1$. The reason behind this result is the following: if the non-deviation condition (16) holds in $\underline{t}+1$ when $A_{t-1} = \underline{t}$, then the analogous non-deviation conditions that would apply to a quality version $q > 1+\rho$ in future periods will always hold true for any $A_{t-1} \geq \underline{t}$. This in turn means that, in any period $t \geq \underline{t}+2$, the action $\varepsilon_{i,q} = 1$ will *strictly dominate* the action $\varepsilon_i = 0$, in a context with $\varepsilon_{j,q} = 1$ for all $j \neq i$.

Lemma 5 addresses the question of whether an economy may sustain an equilibrium with *some* type of innovation effort in the periods that follow $t = \underline{t}+1$. We proceed to study now *which* type of innovation effort takes place in those periods. The next proposition shows that the growth path followed during the horizon $t \geq \underline{t}+1$ will display finite spells with product innovation, alternating with finite spells with process innovation.

Proposition 5 *When* $Q = \{1, 1+\rho, 1+2\rho, ...\}$*, only those economies that satisfy the condition* $\bar{t}(\sigma) \geq \underline{t}(\sigma)$ *will be able to sustain an equilibrium growth path with positive growth in the long run. Along such a growth path, the economy will experience the following growth sequence:*

1. *There is an initial growth phase driven by process innovation effort starting in* $t = 1$ *until period* $t = \underline{t} \geq 1$

2. *There is a second growth phase driven product innovation effort starting in* $t = \underline{t}+1$*, and lasting for a finite number of periods until period* $\widehat{t} \geq \underline{t}+1$*.*

3. *After* $\widehat{t}$*, the growth path exhibits finite spells of growth driven by process innovation effort, alternating with finite spells of growth driven by product innovation effort.*

Proposition 5 shows that economies whose σ turns out to be large enough to make the condition $\bar{t}(\sigma) \geq \underline{t}(\sigma)$ hold true (i.e., $\sigma > \widehat{\sigma}$ as defined in Corollary 1) will be able to exhibit positive income growth in the long run. The growth path of these economies is characterized by an initial phase driven by process innovation effort, followed by a sequence of finite spells of growth driven by product innovation effort and process innovation effort that alternate each other indefinitely. This result showcases the interplay between process and product innovations present in our model. On one side, the quantity expansion brought about by process innovations bolsters the incentives to start investing in quality-upgrading innovations. On the other side, the ensuing quality expansion stemming from product innovations relaxes the inherent tendency of profit prospects from further process innovations to decay. The alternation of equilibria with process and product innovation efforts exploits this feedback loop, and is thus instrumental to keeping income growth alive in the long run.

An interesting implication of Proposition 5 is that the growth path of an economy satisfying $\sigma > \widehat{\sigma}$ will display deterministic growth cycles. In particular, spells where growth rates start to wane owing to the eroding effect of quantity expansion on marginal utility of consumption alternate with periods of higher growth rates in which quality upgrading takes place. In turn, higher growth rates resulting from quality upgrading will reinvigorate growth rates from further quantity expansion. This dynamic mechanism somehow resembles the one featured in Matsuyama (1999), which highlights the alternation of growth cycles driven by capital accumulation (to expand the stock of capital) and innovation activities (to expand the variety of capital goods). In Matsuyama (1999), the decreasing marginal productivity of a fixed variety of capital goods implies that expanding the variety of capital goods is necessary for sustaining long-run growth. In our model, cycles emerge from a different source of interaction: the complementarities of independently meaningful types of innovation effort, whose relative prominence shifts endogenously along the growth path. Notice that an important difference between the models is to do with the origin of the underlying tension leading to growth cycles in each of them. In Matsuyama (1999), cycles are the result of supply-side features of the economy, and hence they are disconnected from the evolution of consumer behavior along the growth path. By contrast, the cycles described by Proposition 5 are crucially linked to the non-homothetic demand structure of the model. In particular, it is owing to such non-homothetic structure that quality upgrades from product innovations keep reviving the demand side of the economy, and thus regenerating the incentives to carry out process innovations.[24]

5.1 Process and Product Innovation along the (Alternating) Growth Path

Proposition 5 shows that an economy featuring long-run growth undergoes an initial growth phase propelled by process innovation effort, followed by a second phase where growth is driven by product innovation effort. After this initial sequence, the economy alternates indefinitely between spells of growth pulled by process innovations and spells of growth pulled by product

[24] Shleifer (1986) and Francois and Lloyd-Ellis (2003) also present models that feature deterministic growth cycles. In those models cycles are the result of the endogenous clustering of innovations in certain moments of time due to the presence of (positive) demand externalities.

innovations. The proposition then leaves still unanswered one crucial question: whether the preeminence of product innovations vis-a-vis process innovations rises along the growth path. The next proposition addresses this question, and shows that this is indeed the case when considering a long enough time horizon.

Proposition 6 *Consider an economy that satisfies the condition $\bar{t}(\sigma) \geq \underline{t}(\sigma)$, and which will therefore be able to sustain positive growth in the long run.*

Let $\mathcal{H}_T$=$\{1, 2, ..., T\}$ denote the set of periods starting in $t = 1$ and ending in $t = T$, and suppose that T is a large number.

Let $\alpha_T \in (0, 1)$ denote the fraction of periods in $\mathcal{H}_T$ that feature an equilibrium with process innovation effort.

Let the time horizon of $\mathcal{H}_T$ be extended by $\Delta \geq 1$ additional periods, and denote the extended set of periods by $\mathcal{H}_{T+\Delta}$=$\{1, 2, ..., T + \Delta\}$. Then, the fraction of periods in $\mathcal{H}_{T+\Delta}$ that feature process innovation effort, $\alpha_{T+\Delta} \in (0, 1)$, will be smaller than in $\mathcal{H}_T$ for Δ large enough. In other words, there exists $\widetilde{\Delta} \geq 1$, such that $\alpha_{T+\Delta} < \alpha_T$ for any $\Delta \geq \widetilde{\Delta}$.

Proposition 6 states that equilibria exhibiting process innovation effort tend to occur less and less often along the growth path. This in turn means that periods where the equilibrium features product innovation effort will tend to be increasingly observed along the growth path. This last result then complements the dynamics described in Proposition 5 regarding the importance and interplay between process and product innovations. It shows that, while both types of innovations fulfil crucial roles in the model to sustain long-run growth, product innovations tend to become increasingly more prominent than process innovations as economies grow richer.

Figure 2 provides a numerical example of the path followed by an economy featuring long-run growth. Each dot in the figure represents one period, and the graph shows the equilibrium sequence from $t = 1$ until $t = 100$. As we can see, there is an initial phase where the economy continuously displays process innovation effort. After that, the economy starts alternating spells with product and process innovation effort, as stipulated by Proposition 5. However, as stated by Proposition 6, the way these equilibria alternate each other does not follow a constant pattern along the growth path. In particular, the equilibrium alternation pattern gets gradually biased towards the equilibria with product innovation effort. In other words, during this alternating sequence, the product innovation equilibrium becomes increasingly ubiquitous, while the process innovation equilibrium becomes more and more sporadic.[25]

[25] The example in Figure 2 is based on values $\sigma = 0.06$ and $\rho = 0.035$. The initial phase where the economy continuously exhibits process innovation effort in equilibrium lasts from $t = 1$ until $t = 20$.

Figure 2: Dynamics with Alternating Spells of Process and Product Innovation

6 Concluding Remarks

We presented a model where the combined impact of process and product innovations steer the economy along a growth path featuring both quantity and quality expansion. At early stages of development, when willingness to pay for quality upgrading is low, growth must be driven by the cost-cutting effect of process innovations. However, an economy cannot rely exclusively on process innovations in order to achieve long-lasting growth, as their benefits tend to decrease as physical production keeps expanding, pushing individuals towards a state of relative satiation. Sustained growth necessitates thus that the economy becomes also able to generate product innovations as it moves along the development path, so as to overturn the tendency towards satiation. In addition, quality-upgrading innovations boost the incentives to keep expanding physical production. Therefore, while process innovations are necessary to turn product innovations sufficiently profitable, product innovations are able to regenerate profit prospects from further process innovations. This implicit feedback loop may keep growth alive in the long run.

Our model has restricted the consumption space to a very specific case: one single final good available in different quality versions, which are all perfect substitutes among each other. One important type of innovation effort that our model has then ruled out is that one that leads to a *horizontal* expansion in the set of final goods, as in Judd (1985), Romer (1990), and Grossman and Helpman (1991c, Ch.3), Young (1993). In principle, these types of innovations may also be able to keep growth alive in the long run. In particular, as profit prospects from cost-cutting innovations dwindle owing to decreasing marginal utility in a *given* good category, individuals may at some point find it worthwhile to introduce a completely *new* good category. This new final good would offer initially large profit prospects from process innovations, which would tend to diminish with subsequent rounds of it. We see this mechanism leading to a horizontal expansion of the set of consumption goods as complementary to the interplay between quantity and quality expansion studied by our model. Certainly, a model in which growth features a simultaneous expansion in quantity, quality and variety of consumption, with positive feedbacks between all three dimensions, could yield a more encompassing description of growth in mature economies, and we see this as an appealing avenue of future research.

One other limitation of the model is that it has stuck to a framework without income inequality. The presence of non-homothetic preferences means that income distribution matters for the aggregate demand structure of the economy. Within our model, some (initial) inequality may turn out to increase the chances that an economy is able to feature sustained growth in the long run. In particular, the presence of rich consumers may speed up the introduction of quality-upgrading innovations, which in turn switches on the needed transition to a path of long-run growth driven by the interaction between process and product innovations.

Finally, our model studies the case of a closed economy in autarky. An interesting question that we cannot thus address here is whether our framework, adapted to include open economies and trade, could possibly lead to some sort of international specialization of innovation effort by type. In particular, in the presence of trade costs and non-homothetic preferences, we may obtain dynamics where process innovation effort tends to gradually move to lower-income economies, while richer economies increasingly specialize in generating product innovations. This result would be somehow reminiscent of the Linder's hypothesis of quality specialization in trade, and could therefore provide an explanation of that theory originated from a fully-fledged endogenous growth model. Similarly, along a sustained growth path, such dynamics may feature product life cycles resembling those in Vernon (1966), where product improvements would be generated in richer economies, while lower-income economies would gradually develop a comparative advantage in generating subsequent process innovations.

Appendix A: Proofs

Proof of Lemma 1. A household in t will optimally set $\varepsilon_t^* = 1$ iff the utility obtained from consuming $1 + \sigma A_{t-1} + \sigma$ units of the baseline quality good is strictly greater than the utility derived from consuming $1 + \sigma A_{t-1}$ units of it plus the utility of leisure, η. Using (9) and (10), together with the utility function (4) when $Q_t = \{1\}$, condition (11) obtains. ■

Proof of Proposition 1. The proof follows immediately from the derivations in the main text, together with the fact that (1) implies that after a sequence of t consecutive periods featuring an equilibrium with positive process innovation effort we have that $A_t = t$. ■

Proof of Lemma 2. We carry out the proof in three separate steps.

Step 1) $P_t > (1+\rho)\, Y_t^{\rho/(1+\rho)}$ cannot hold in equilibrium.

Using (4) we can observe that the utility obtained by a household in t if they choose to consume the version of the good with quality $q = 1+\rho$ is given by:

$$U_t(q = 1+\rho) = \ln \left[(1+\rho)\, \frac{Y_t}{P_t} \right]^{1+\rho}. \tag{27}$$

Instead, if they choose to consume the baseline quality version, they would obtain:

$$U_t(q = 1) = \ln (Y_t). \tag{28}$$

Hence, comparing (27) and (28), we can observe that $P_t > (1+\rho)\, Y_t^{\rho/(1+\rho)}$ implies $U_t(q = 1+\rho) < U_t(q = 1)$, and therefore no household would consume the higher-quality version.

Step 2) $P_t < (1+\rho)\, Y_t^{\rho/(1+\rho)}$ cannot hold in equilibrium.

Suppose in equilibrium firms require researchers to exert product innovation effort. Suppose also that $P_t = \widetilde{P}_t < (1+\rho)\, Y_t^{\rho/(1+\rho)}$. Since an equilibrium must necessarily satisfy the zero-profit condition for firms, it must be the case that researchers are paid a salary

$$\widetilde{\pi}_t = \widetilde{P}_t \frac{1+\sigma A_{t-1}}{1+\rho} - (1+\sigma A_{t-1})$$

for the effort time. Suppose now that some firm decides to offer researchers a salary $\widehat{\pi}_t$ for their effort time, where

$$\widehat{\pi}_t \equiv \left(\widetilde{P}_t + \widehat{\varepsilon} \right) \frac{1+\sigma A_{t-1}}{1+\rho} - (1+\sigma A_{t-1}), \quad \text{and} \quad \widehat{\varepsilon} > 0.$$

This firm would then attract all researchers alive in t. Furthermore, this firm could charge a price $P_t' \equiv \widetilde{P}_t + \varepsilon' < (1+\rho)\, Y_t^{\rho/(1+\rho)}$, where $\varepsilon' > \widehat{\varepsilon} > 0$, for the higher-quality version of the final good, obtaining positive (expected) profits. As a consequence, a situation where a firm charges a price $P_t < (1+\rho)\, Y_t^{\rho/(1+\rho)}$ for the higher-quality version while it also satisfies the zero profit condition cannot arise in equilibrium.

Step 3) Using again (27) and (28), we can first observe that when $P_t = (1+\rho)\, Y_t^{\rho/(1+\rho)}$ households alive in t are indifferent between the baseline quality and the higher-quality version. Moreover, when (13) holds, there exist no profitable deviation to any firm. In particular, in order to outcompete a firm whose strategy is characterized by (14) and (13), another firm should either offer the higher-quality version for a lower price or, alternatively, offer researchers a higher salary while keeping $P_t = (1+\rho)\, Y_t^{\rho/(1+\rho)}$ (since $P_t > (1+\rho)\, Y_t^{\rho/(1+\rho)}$ cannot hold in equilibrium, as shown by *Step 1*). Both deviations, however, lead to a loss for firms. ■

Proof of Lemma 3. Using (13), we obtain that when all researchers alive in t exert product innovation effort:

$$Y_t = P_t \frac{1+\sigma A_{t-1}}{1+\rho}. \tag{29}$$

Replacing (29) into (14) yields $P_t = (1+\rho)\left[P_t\,(1+\sigma A_{t-1})\,(1+\rho)^{-1}\right]^{\frac{\rho}{1+\rho}}$, from where we may solve for P_t and obtain:

$$P_t = (1+\rho)\,(1+\sigma A_{t-1})^{\rho}. \tag{30}$$

Lastly, plugging (30) into (29) yields (15). ■

Proof of Lemma 4. Part *i*). First of all, notice that equation (15) implies that when all R_i alive in t exert product innovation effort, $Y_t = (1+\sigma A_{t-1})^{1+\rho}$. In this situation, the level of utility achieved by any generic household i alive in t is given by

$$U_{i,t}(\varepsilon_{i,t,q} = 1 |\, \varepsilon_{t,q} = 1) = \ln\left[(1+\rho)\,\frac{(1+\sigma A_{t-1})^{1+\rho}}{P_t}\right]^{1+\rho}. \tag{31}$$

Using the expression in (14), together with $Y_t = (1+\sigma A_{t-1})^{1+\rho}$, (31) yields:

$$U_{i,t}(\varepsilon_{i,t,q} = 1 |\, \varepsilon_{t,q} = 1) = (1+\rho)\ln(1+\sigma A_{t-1}). \tag{32}$$

Suppose now this household would deviate from $\varepsilon_{i,t,q} = 1$ to $\varepsilon_{i,t} = 0$. In this case, $Y_{i,t}(\varepsilon_{i,t} = 0) = w_t = 1+\sigma A_{t-1}$. Recall that the price (14) leaves indifferent a household with $Y_t = (1+\sigma A_{t-1})^{1+\rho}$ between the two versions of the consumption good. Since, $1+\sigma A_{t-1} < (1+\sigma A_{t-1})^{1+\rho}$, it must then be the case that, if setting $\varepsilon_{i,t} = 0$, household i will then strictly prefer to consume the baseline quality rather than (the more expensive) higher-quality version. Moreover, when $w_t = 1+\sigma A_{t-1}$, there will always be a firm willing to offer the baseline quality version, as it would break even by doing so. Hence, when a generic household i alive in t sets $\varepsilon_{i,t} = 0$, within a context where the rest are setting $\varepsilon_{t,q} = 1$, it achieves

$$U_{i,t}(\varepsilon_{i,t} = 0 | \, \varepsilon_{t,q} = 1) = \ln(1 + \sigma A_{t-1}) + \eta \tag{33}$$

Finally, comparing (32) and (33), condition (16) ensures that $U_{i,t}(\varepsilon_{i,t,q} = 1 | \, \varepsilon_{t,q} = 1) > U_{i,t}(\varepsilon_{i,t} = 0 | \, \varepsilon_{t,q} = 1)$, completing the proof.

Part *ii*) Notice first that a firm must pay the same salary regardless of the type of R&D effort exerted by the researcher. Hence, denoting by ϖ the salary paid to a researcher, we have that the expected return of a process innovation is equal to $\sigma - \varpi$. On the other hand, when all R_i members are exerting product innovation effort, using (13) and (15), it follows that the expected return of a product innovation is: $(1+\sigma A_{t-1})^{1+\rho} - (1+\sigma A_{t-1}) - \varpi$. As a result when (17) holds, no firm will prefer to request set $\varepsilon_{i,t,p} = 1$ from R_i instead of $\varepsilon_{i,t,q} = 1$. ■

Proof of Proposition 2. Part *i*) The proof that when (16) and (17) hold there exists an equilibrium in t with $\varepsilon^*_{t,q}$ follows directly from Lemma 4.

Part *ii*) We prove this part of the proposition in three separate steps.

Step 1) When (17) holds, an equilibrium with $\varepsilon^*_{t,p} = 1$ does not exist.
Proof. Using (14) and (13), we can observe that a necessary condition for an equilibrium with $\varepsilon^*_{t,p} = 1$ to exist is that: $Y_t^{(1+\rho)/\rho}(1 + \sigma A_{t-1}) - (1 + \sigma A_{t-1}) < \sigma$. This leads to:

$$Y_t < \left(\frac{1 + \sigma + \sigma A_{t-1}}{1 + \sigma A_{t-1}} \right)^{\frac{1+\rho}{\rho}}. \tag{34}$$

Consider then the case when all researchers set $\varepsilon_{t,p} = 1$. In this situation Y_t is given by (9). Plugging this value into the LHS of (34) leads after some simple algebra to $(1 + \sigma + \sigma A_{t-1}) > (1 + \sigma A_{t-1})^{1+\rho}$, contradicting (17).

Step 2) When (17) holds and (18) does not, an equilibrium with $\varepsilon^*_t = 0$ does not exist.
Proof. Notice first that (18) not holding means that (11) holds true. Therefore, when (18) fails to hold no household i alive in t would thus set $\varepsilon_{i,t} = 0$ in equilibrium. Moreover, the fact that (17) holds in turn implies that the unique equilibrium in this case must feature $\varepsilon^*_{t,q} = 1$.

Step 3) When, alongside (16) and (17), also both (18) and (19) hold true, an equilibrium with $\varepsilon^*_t = 0$ also exists.

When all households alive in t set $\varepsilon_t = 0$, equation (10) applies, and thus $Y_t = 1 + \sigma A_{t-1}$. In this situation, in case a product innovation were created as a result of a deviation by a generic household i to $\varepsilon_{i,t,q} = 1$, the price of $q = 1 + \rho$ would be $P_t = (1 + \rho)(1 + \sigma A_{t-1})^{\rho/(1+\rho)}$. Thus, the expected return generated by the product innovation blueprint created when household i deviates to $\varepsilon_{i,t,q} = 1$ would be equal to $\pi^q_{i,t} = (1 + \sigma A_{t-1})^{(1+2\rho)/(1+\rho)} - (1 + \sigma A_{t-1})$, which in equilibrium should be equal as well to the salary paid to R_i. As a consequence, by deviating to $\varepsilon_{i,t,q} = 1$ in a context where all other households set $\varepsilon_t = 0$, household i's total earnings would be $Y_{i,t} = (1 + \sigma A_{t-1})^{(1+2\rho)/(1+\rho)}$. Notice now that since R_i has measure zero, after his unilateral deviation to $\varepsilon_{i,t,q} = 1$, the probability that the researcher ends up generating the successful product innovation blueprint is actually zero. Therefore, when household i is contemplating whether deviate unilaterally to $\varepsilon_{i,t,q} = 1$, they are also aware that (almost surely) the only version that will be offered by firms will be the baseline quality, $q = 1$. As a result, the utility that household i expects to obtain should they deviate unilaterally to $\varepsilon_{i,t,q} = 1$ is given by:

$$U_{i,t}(\varepsilon_{i,t,q} = 1 |\, \varepsilon_t = 0) = \ln{(1 + \sigma A_{t-1})}^{(1+2\rho)/(1+\rho)}. \tag{35}$$

On the other hand, by sticking to $\varepsilon_{i,t} = 0$, household i would obtain:

$$U_{i,t}(\varepsilon_{i,t} = 0 |\, \varepsilon_t = 0) = \ln{(1 + \sigma A_{t-1})} + \eta. \tag{36}$$

An equilibrium where $\varepsilon^*_t = 0$ will exist if $U_{i,t}(\varepsilon_{i,t,q} = 1 |\, \varepsilon_t = 0) \leq U_{i,t}(\varepsilon_{i,t} = 0 |\, \varepsilon_t = 0)$. Therefore, using (35) and (36), we can obtain (19). This completes the proof that when both (18) and (19) are verified by an economy that also satisfies (16) and (17), then two equilibria exist (among the class of symmetric Nash equilibria in pure strategies): $\varepsilon^*_{t,q} = 1$ and $\varepsilon^*_t = 0$. ■

Proof of Proposition 3. Part i) From Proposition 1, it follows that the economy will keep growing through process innovation effort until $t = \bar{t}(\sigma)$, and reach a level of $A_{\bar{t}} = \bar{t}(\sigma)$ in that period. Consider now what happens in $t = \bar{t}(\sigma) + 1$. We know from Proposition 1 that process innovation will stop in that period. Also, the fact that $\underline{t}(\sigma) > \bar{t}(\sigma)$ means that $\bar{t}(\sigma) \leq \underline{t}(\sigma) - 1$. Furthermore, from Definition 2, it follows that $\underline{A}(\sigma) > \underline{t}(\sigma) - 1$. Therefore, $\bar{t}(\sigma) < \underline{A}(\sigma)$, in turn implying that there will not be an equilibrium with product innovation either in $t = \bar{t}(\sigma) + 1$. Finally, in the absence of any type of innovation effort, the same situation will repeat itself in $t = \bar{t}(\sigma) + 2$, and thereafter.

Part ii) Given the results in Proposition 1, we can observe that the economy will keep growing through process innovation effort until $t = \underline{t}(\sigma)$, and reach a level of $A_{\underline{t}} = \underline{t}(\sigma)$ in that period. Consider now what happens in period $t = \underline{t}(\sigma) + 1$. Using again Definition 2, $\underline{t}(\sigma) > \underline{A}(\sigma)$. Therefore, the conditions in Lemma 4 must hold true in $t = \underline{t}(\sigma) + 1$, and the economy will therefore exhibit an equilibrium with product innovation effort in that period. Finally, owing to Proposition 2, in this situation the economy cannot possibly exhibit an equilibrium in $t = \underline{t}(\sigma) + 1$ with process innovation effort, which completes the proof. ■

Proof of Proposition 4. From Assumption 2, we can observe that firms active in $t = \underline{t} + 2$ will inherit a technology that will allow them to produce $(1 + \sigma\underline{t})/(1 + \rho)$ units of the quality version $q = 1 + \rho$ with one unit of labor. Letting $P_{\underline{t}+2}$ denote the price of the quality version $q = 1 + \rho$ in period $t = \underline{t} + 2$, it then follows that the equilibrium wage in that period will be $w_{\underline{t}+2} = P_{\underline{t}+2}(1 + \sigma\underline{t})/(1 + \rho)$, which using the result in (14) leads to:

$$w_{\underline{t}+2} = Y_{\underline{t}+2}^{\rho/(1+\rho)}(1 + \sigma\underline{t}). \tag{37}$$

Notice now that since $Y_{\underline{t}+2}^{\rho/(1+\rho)} > 1$, no firm active in $t = \underline{t} + 2$ will, in equilibrium, offer the baseline quality version (if one of these firms did so, it would make a loss). As a result, in $t = \underline{t} + 2$ the only quality version that will be actively offered in the market is $q = 1 + \rho$.

Consider now the effect of a process innovation in period $t = \underline{t}+2$. This would allow the firm that implements the process innovation to produce $(1 + \sigma\underline{t} + \sigma)/(1 + \rho)$ units of the quality version $q = 1 + \rho$ with one unit of labor. As a consequence, the expected return of a process innovation blueprint will be:

$$\pi_{\underline{t}+2}^{p} = Y_{\underline{t}+2}^{\rho/(1+\rho)}\sigma. \tag{38}$$

Using (37) and (38), we can then obtain for a household i alive in $t = \underline{t} + 2$:

$$Y_{i,\underline{t}+2}(\varepsilon_{i,\underline{t}+2}^{p} = 1) = Y_{\underline{t}+2}^{\rho/(1+\rho)}(1 + \sigma\underline{t} + \sigma), \tag{39}$$

$$Y_{i,\underline{t}+2}(\varepsilon_{i,\underline{t}+2}^{p} = 0) = Y_{\underline{t}+2}^{\rho/(1+\rho)}(1 + \sigma\underline{t}). \tag{40}$$

Lastly, using (39), (40) and the utility function (4), bearing in mind $P_{\underline{t}+2} = (1+\rho)Y_{\underline{t}+2}^{\rho/(1+\rho)}$, we can observe that households in $t = \underline{t} + 2$ will set $\varepsilon_{\underline{t}+2}^{p} = 1$ if and only if (25) holds true. ∎

Proof of Lemma 5. We carry out this proof by showing that when $\bar{t}(\sigma) \geq \underline{t}(\sigma)$ holds, in a context where the set Q is unbounded, then for any generic household i alive in $t > \underline{t}$ the action $\varepsilon_{i,t,q} = 1$ strictly dominates $\varepsilon_{i,t} = 0$, when all other $j \neq i$ alive in t set $\varepsilon_{j,t,q} = 1$.

Step 1. $t = \underline{t} + 1$: The fact that when $\bar{t}(\sigma) \geq \underline{t}(\sigma)$ holds, $\varepsilon_{i,t,q} = 1$ strictly dominates $\varepsilon_{i,t} = 0$ when all other $j \neq i$ alive in t set $\varepsilon_{j,t,q} = 1$ follows directly from Proposition 3.

Step 2. Generalization of Lemma 2 when Q is unbounded above: When we let Q=$\{1, 1+\rho, 1+2\rho, ...\}$, it follows that the expected return of the product innovation blueprint designed by a generic R_i alive in t will be:

$$\pi_{i,t}^{q} = P_t(q_t)\frac{1 + \sigma A_{t-1}}{q_t} - w_t, \tag{41}$$

where $P_t(q_t)$ is the price of the (newly designed) quality version $q_t \in Q$. To compute the equilibrium value of $P_t(q_t)$, notice that, based on (4), this will follow from the condition

$$\ln\left(q_t\frac{Y_t}{P_t(q_t)}\right)^{q_t} \geq \ln(Y_t), \tag{42}$$

from where we can obtain

$$P_t(q_t) = q_t \, Y_t^{(q_t - 1)/q_t} \tag{43}$$

when (42) holds with equality. Lastly, the fact that all firms active in t inherit a technology that allows producing $(1 + \sigma A_{t-1}) / q_{t-1}$ units of the quality version $q_{t-1} \in Q$ with one unit of labor in turn implies that:

$$w_t = P_t(q_{t-1}) \frac{1 + \sigma A_{t-1}}{q_{t-1}} = Y_t^{(q_{t-1} - 1)/q_{t-1}} \left(1 + \sigma A_{t-1}\right), \tag{44}$$

where $q_{t-1} = q_t - \rho$ when all R_i alive in t are hired to exert product innovation effort.

Step 3. Generalization of Lemma 3 when Q is unbounded above: If all households alive in t set $\varepsilon_{t,q} = 1$, using (41), (43) and (44), we obtain:

$$Y_t(\varepsilon_{t,q} = 1) = \left(1 + \sigma A_{t-1}\right)^{q_t}. \tag{45}$$

Step 4. $t \geq \underline{t} + 2$: Due to Proposition 3, when $\bar{t}(\sigma) \geq \underline{t}(\sigma)$ holds, we must have that $A_{t-1} \geq \underline{t}$ and $q_{t-1} \geq 1 + \rho$. Suppose also that $\varepsilon_{t,q} = 1$ in each period $t \geq \underline{t} + 2$. Then, using (45):

$$Y_t(\varepsilon_{t,q} = 1) = \left[1 + \sigma \left(\underline{t} + \delta\right)\right]^{q_t}, \tag{46}$$

where $\delta \geq 0$. Using next the utility function (4), together with (46) and (43), we may obtain:

$$U_{i,t}(\varepsilon_{i,t,q} = 1 | \, \varepsilon_{t,q} = 1) = q_t \ln \left[1 + \sigma \left(\underline{t} + \delta\right)\right], \tag{47}$$

which denotes the level of utility achieved by a generic household i alive in $t \geq \underline{t} + 2$ when they stick to the choice $\varepsilon_{i,t,q} = 1$, given that all other households set $\varepsilon_{t,q} = 1$. On the other hand, if in such same circumstance i deviates to $\varepsilon_{i,t} = 0$, they will achieve:

$$U_{i,t}(\varepsilon_{i,t} = 0 | \, \varepsilon_{t,q} = 1) = q_{t-1} \ln \left[1 + \sigma \left(\underline{t} + \delta\right)\right] + \eta = (q_t - \rho) \ln \left[1 + \sigma \left(\underline{t} + \delta\right)\right] + \eta. \tag{48}$$

Comparing (47) and (48), we obtain:

$$U_{i,t}(\varepsilon_{i,t,q} = 1 | \, \varepsilon_{t,q} = 1) > U_{i,t}(\varepsilon_{i,t} = 0 | \, \varepsilon_{t,q} = 1) \iff \rho \ln \left[1 + \sigma \left(\underline{t} + \delta\right)\right] > \eta. \tag{49}$$

Finally, since $\delta \geq 0$, it follows that when (16) holds in $t = \underline{t} + 1$ (which is when $A_{t-1} = \underline{t}$), then (49) will hold true in any period $t \geq \underline{t} + 2$. Bearing in mind that $\rho \ln (1 + \sigma \underline{t}) > \eta$ is a necessary condition for $\bar{t}(\sigma) \geq \underline{t}(\sigma)$ to hold, this last step completes the proof that $\varepsilon_{i,t,q} = 1$ strictly dominates $\varepsilon_{i,t} = 0$, when all other households alive in t set $\varepsilon_{t,q} = 1$. ■

Proof of Proposition 5.

Part 1. The fact that there is an initial growth phase, between $t = 1$ and $t = \underline{t} \geq 1$ follows from Proposition 3.

Part 2. The fact that there is a second growth phase starting in $t = \underline{t} + 1$ that is driven by product innovation effort also follows from Proposition 3. Next, to prove that this growth phase lasts for a finite number of periods (i.e., it lasts until $t = \widehat{t} \geq \underline{t} + 1$) we proceed by contradiction. Suppose that the only type of innovation effort undertaken during $t > \underline{t} + 1$ is in product innovation. In this case, we would have $A_{t-1} = \underline{t}$ for all $t > \underline{t} + 1$. In addition, the highest quality version available in periods $t > \underline{t}+1$ would be $q_t = 1+(t-\underline{t})\rho$. As a consequence, when $\varepsilon_{t,q} = 1$ for all $t > \underline{t} + 1$, the expected return generated by a product innovation blueprint (net of wages, w_t) will be

$$\pi_t^q(\widetilde{\varepsilon}_{q,t>\underline{t}+1}) = (1 + \sigma\underline{t})^{1+(t-\underline{t})\rho} - w_t, \tag{50}$$

where we use $\widetilde{\varepsilon}_{q,t>\underline{t}+1}$ to denote the *hypothetical* path in which $\varepsilon_{t,q} = 1$ for all $t > \underline{t} + 1$. For this to be an equilibrium, $\pi_t^q(\widetilde{\varepsilon}_{q,t>\underline{t}+1})$ should be larger than the expected return obtained when shifting a generic researcher R_i alive in t to setting $\varepsilon_{i,t,p} = 1$, which would yield:

$$\pi_t^p(\varepsilon_{i,t,p} = 1 \,|\widetilde{\varepsilon}_{q,t>\underline{t}+1}) = Y_t(\widetilde{\varepsilon}_{q,t>\underline{t}+1})^{(t-1-\underline{t})\rho/[1+(t-1-\underline{t})\rho]} (1 + \sigma\underline{t} + \sigma) - w_t. \tag{51}$$

Comparing (50) and (51), we can observe that $\pi_t^q(\widetilde{\varepsilon}_{q,t>\underline{t}+1}) > \pi_t^q(\widetilde{\varepsilon}_{q,t>\underline{t}+1})$ requires $(1 + \sigma\underline{t})^{1+(t-\underline{t})\rho} > (1 + \sigma\underline{t} + \sigma)^{1+[(t-1)-\underline{t}]\rho}$, which will fail to hold when t becomes sufficiently large (i.e., when t departs sufficiently from $\underline{t}$).

Part 3. Lastly, to prove that after $t = \widehat{t}$ the economy will be able to sustain positive growth forever by alternating finite spells where the equilibrium features process innovation effort with finite spells where the equilibrium features product innovation effort, we proceed by again contradiction, while bearing in mind the result in Lemma 5.Consider first the case of a hypothetical economy that for all periods $t \geq t'$ features process innovation effort in equilibrium, where we let $t' > \widehat{t}$. In that case, we will have $A_t = t - (q_{t'-1} - 1)/\rho$ and $q_t = q_{t'-1}$, for all $t \geq t'$. The expected returns of a process innovation blueprint in given period $t > t'$ will be

$$\pi_t^p(\widetilde{\varepsilon}_{p,t\geq t'}) = \{1 + \sigma [t - (q_{t'-1} - 1)/\rho]\}^{q_{t'-1}} - w_t, \tag{52}$$

where $\widetilde{\varepsilon}_{p,t\geq t'}$ denotes the *hypothetical* growth path in which $\varepsilon_{t,p} = 1$ for all $t \geq t'$. For this to be an equilibrium, $\pi_t^p(\widetilde{\varepsilon}_{p,t>\underline{t}+1})$ should be larger than the expected return obtained when shifting a generic researcher R_i alive in t to setting $\varepsilon_{i,t,q} = 1$, which would yield

$$\pi_t^q(\varepsilon_{i,t,q} = 1 \,|\widetilde{\varepsilon}_{p,t\geq t'}) = Y_t(\widetilde{\varepsilon}_{p,t\geq t'})^{(q_{t'-1}+\rho-1)/(q_{t'-1}+\rho)} \{1 + \sigma [t - 1 - (q_{t'-1} - 1)/\rho]\} - w_t. \tag{53}$$

Comparing (52) and (53), it follows that $\pi_t^p(\widetilde{\varepsilon}_{p,t\geq t'}) > \pi_t^q(\varepsilon_{i,t,q} = 1 \,|\widetilde{\varepsilon}_{p,t\geq t'})$ requires

$$\{1 + \sigma\left[t - (q_{t'-1} - 1)/\rho\right]\}^{q_{t'-1}} > \{1 + \sigma\left[t - 1 - (q_{t'-1} - 1)/\rho\right]\}^{q_{t'-1}+\rho}$$

which will fail to hold when t becomes sufficiently large. As a consequence, the economy cannot possibly sustain an equilibrium growth path with only process innovation effort over an infinitely long sequence of consecutive periods.

Consider next the case of a hypothetical economy that for all periods $t \geq t'$ features product innovation effort in equilibrium, where again we let $t' > \widehat{t}$. In this case, $A_{t-1} = (t'-1) - (q_{t'-1} - 1)/\rho$ and $q_t = q_{t'-1} + (t - t' + 1)\rho$, for all $t \geq t'$. These results in turn imply that in a generic period $t \geq t'$ we will have:

$$\pi_t^q(\widetilde{\varepsilon}_{q,t\geq t'}) = \{1 + \sigma\left[(t'-1) - (q_{t'-1} - 1)/\rho\right]\}^{q_{t'-1}+(t-t'+1)\rho} - w_t, \tag{54}$$

where we use $\widetilde{\varepsilon}_{q,t\geq t'}$ to denote the *hypothetical* path in which $\varepsilon_{t,q} = 1$ for all $t \geq t'$. For this to be an equilibrium, $\pi_t^q(\widetilde{\varepsilon}_{q,t>\underline{t}+1})$ should be larger than the expected return obtained when shifting a generic researcher R_i alive in t to setting $\varepsilon_{i,t,p} = 1$, which would yield:

$$\pi_t^p(\varepsilon_{i,t,p} = 1 \,|\widetilde{\varepsilon}_{q,t\geq t'}) = Y_t(\widetilde{\varepsilon}_{q,t\geq t'})^{[q_{t'-1}+(t-t')\rho-1]/[q_{t'-1}+(t-t')\rho]}\{1 + \sigma\left[t' - (q_{t'-1} - 1)/\rho\right]\} - w_t. \tag{55}$$

Comparing (54) and (55), it follows that $\pi_t^q(\widetilde{\varepsilon}_{q,t\geq t'}) > \pi_t^p(\varepsilon_{i,t,p} = 1 \,|\widetilde{\varepsilon}_{q,t\geq t'})$ requires

$$\{1 + \sigma\left[(t'-1) - (q_{t'-1} - 1)/\rho\right]\}^{q_{t'-1}+(t-t'+1)\rho} > \{1 + \sigma\left[t' - (q_{t'-1} - 1)/\rho\right]\}^{q_{t'-1}+(t-t')\rho}$$

which will fail to hold when t becomes sufficiently large. As a result, the economy cannot possibly sustain an equilibrium growth path with only product innovation effort over an infinitely long sequence of consecutive periods.

The previous two contradictions imply thus that there cannot exist an equilibrium growth path featuring either infinitely long spells of process innovation effort or infinitely long spells of product innovation effort. Lemma 5 stipulates that an economy satisfying the condition $\bar{t}(\sigma) \geq \underline{t}(\sigma)$ is always able to sustain an equilibrium with some type of innovation effort. Hence, it must be the case that the growth path followed by an economy satisfying $\bar{t}(\sigma) \geq \underline{t}(\sigma)$ will exhibit finite spells where researchers exert process innovation effort in equilibrium, alternating with finite spells where they exert product innovation effort in equilibrium. ■

Proof of Proposition 6. When T is a large number and the fraction of periods in $\mathcal{H}_T$ featuring an equilibrium with $\varepsilon^*_{t,p} = 1$ is given by α_T, we have that:

$$q_T = 1 + (1 - \alpha_T)\,T\rho \tag{56}$$

$$A_T = \alpha_T\,T. \tag{57}$$

Using (56) and (57), we can observe that the equilibrium in $T+1$ will feature $\varepsilon^*_{T+1,p} = 1$ if $(1+\alpha_T T\sigma)^{1+(1-\alpha_T)T\rho+\rho} < (1+\alpha_T T\sigma+\sigma)^{1+(1-\alpha_T)T\rho}$ holds true, while the economy will have an equilibrium with $\varepsilon^*_{T+1,q} = 1$ when $(1+\alpha_T T\sigma)^{1+(1-\alpha_T)T\rho+\rho} > (1+\alpha_T T\sigma+\sigma)^{1+(1-\alpha_T)T\rho}$. The previous two inequalities can be combined, to obtain

$$\begin{aligned} \varepsilon^*_{T+1,q} = 1 \quad &\text{if} \quad \Gamma(\alpha_T, T) > \Upsilon(\alpha_T, T), \\ \varepsilon^*_{T+1,p} = 1 \quad &\text{if} \quad \Gamma(\alpha_T, T) < \Upsilon(\alpha_T, T). \end{aligned} \tag{58}$$

where

$$\Gamma(\alpha_T, T) \equiv 1 + \frac{\rho}{1+(1-\alpha_T)\,T\rho} \quad \text{and} \quad \Upsilon(\alpha_T, T) \equiv \frac{\ln(1+\alpha_T T\sigma+\sigma)}{\ln(1+\alpha_T T\sigma)}. \tag{59}$$

Notice, first, from (59) that $\partial\Gamma(\cdot)/\partial\alpha_T > 0, \partial\Upsilon(\cdot)/\partial\alpha_T < 0$, $\partial\Gamma(\cdot)/\partial T < 0$, and $\partial\Upsilon(\cdot)/\partial T < 0$. It can also be shown (see proof in Appendix D) that, there exists a finite value $\widehat{T}(\alpha_T)$, such that $\Gamma(\alpha_T, \widehat{T}(\alpha_T)) = \Upsilon(\alpha_T, \widehat{T}(\alpha_T))$, and $\Gamma(\alpha_T, T) < \Upsilon(\alpha_T, T)$ for $T < \widehat{T}$ while $\Gamma(\alpha_T, T) > \Upsilon(\alpha_T, T)$ for $T > \widehat{T}$.

Suppose now that after extending the horizon from T to $T+\Delta$, the fraction of periods in which the equilibrium exhibits $\varepsilon^*_{t,p} = 1$ remains equal to α_T. Then, there will be some $\widetilde{\Delta}$ such that for all $\Delta > \widetilde{\Delta}$, we will have that $\Gamma(\alpha_T, T+\Delta) > \Upsilon(\alpha_T, T+\Delta)$. As a consequence, the fraction of periods in which $\varepsilon^*_{t,p} = 1$ cannot remain indefinitely equal to α_T. Furthermore, since $\partial\Gamma(\cdot)/\partial\alpha_T > 0$ and $\partial\Upsilon(\cdot)/\partial\alpha_T < 0$, it must then be the case that $\alpha_{T+\Delta} < \alpha_T$ for $\Delta > \widetilde{\Delta}$, where $\widetilde{\Delta}$ is a finite positive value. ■

Appendix B: Complementary Correlations for Table 1

The correlations displayed in Table 1 should only be taken as motivational evidence for our theory and model, and by no means we intend to present them as showing a causal effect of any sort. Nevertheless, it is important to show that these simple correlations are robust, and at the same time that they do not simply capture other country-level variables correlated with GDP per head. Here we provide some of these robustness checks.

Columns (1) - (3) of Table B.1 show the results of our previous regressions in columns (2) - (4) of Table 1 when using the logarithm of GDP per head, instead of the level. As we can readily see, this has essentially no qualitative effect on our previous results.

Next, recall that column (2) of Table 1 considered only those firms that have introduced process innovations, while column (3) considered only those firms that have introduced product innovations. However, some of the firms used for column (2) have also introduced product innovations, while some of the firms used for column (3) have also introduced process innovations. Columns (4) - (6) of Table B.1 show the results of our previous columns (2) - (4) of Table 1 with the following modification: *i*) in column (4) we consider *only* those firms that have introduced process innovations but have *not* introduced product innovations; *ii*) in column (5) we consider *only* those firms that have introduced product innovations but have *not* introduced process innovations. The results follow a similar pattern as those in Table 1, but are even more pronounced than before (in fact, the correlation between GDP per head and the ratio of firms doing only process innovation and not doing product innovation, becomes now insignificant).

Table B.2 introduces in columns (1) - (3) total GDP as an additional independent variable. This would control for the presence of an aggregate size effect of the home market, which could differently affect investment in process and in product innovation. As it can be readily observed, all the correlations previously presented in Table 1 remain essentially intact, and the coefficient associated to total GDP is never significantly different from zero.

Finally, the results in Table 1 could alternatively be driven by a supply-side mechanism, rather than by our proposed income-dependent demand for quality upgrading. One such supply-side mechanism could be the result of disparities in the endowments of skilled labor across countries (for instance, if the stock of human capital could be more important for the generation of product innovations than for process innovations), since skilled labor endowments tend to correlate positively with GDP per capita. In order to see whether such type of supply-side mechanism is the one behind the correlations in Table 1, we also include in columns (4) - (6) the average educational attainment in each country in the sample. As we can see, none of the previous results in Table 1 are significantly altered by this. Another possible supply-side

Table B.1

	Dependent Variable					
	ratio firms doing proc. innovation	ratio of firms prod. innovation	ratio of prod. to proc. innov.	ratio firms doing ONLY process innov.	ratio firms doing ONLY product innov.	ratio of ONLY prod. to ONLY proc. innov.
Log GDP per head	0.072**	0.112***	0.150**			
	(0.026)	(0.024)	(0.073)			
GDP per head				0.004	0.024***	0.272***
				(0.005)	(0.004)	(0.074)
R-squared	0.21	0.44	0.13	0.03	0.55	0.33
Number countries	30	30	30	30	30	30

Standard errors reported in parentheses. Column (4) uses as dependent variable that ratio of firms responding 'Yes' to doing 'process innovation' and 'No' to doing 'product innovation' Column (5) uses as dependent variable that ratio of firms responding 'Yes' to doing 'product innovation' and 'No' to doing 'process innovation'. Column (6) uses the ratio between the dependent variables in column (5) and in column (4). * significant 10%; ** significant 5%; *** significant 1%.

Table B.2

	Dependent Variable								
	ratio firms proc. innov.	ratio firms prod. innov.	ratio of prod. to proc. innov.	ratio firms proc. innov.	ratio firms prod. innov.	ratio of prod. to proc. innov.	ratio firms proc. innov.	ratio firms prod. innov.	ratio of prod. to proc. innov.
GDP per head	0.020*	0.038***	0.071***	0.022*	0.038***	0.064**	0.012	0.034***	0.078**
	(0.010)	(0.009)	(0.026)	(0.011)	(0.010)	(0.026)	(0.013)	(0.012)	(0.031)
Total GDP	0.00000246	0.0000131	0.0000323	0.00000158	0.0000131	0.000035	0.00000519	0.0000102	0.0000451
	(0.0000188)	(0.0000166)	(0.0000466)	(0.0000186)	(0.0000169)	(0.000045)	(0.0000189)	(0.0000176)	(0.0000472)
Years of Schooling				-0.0132	-0.0007	0.0409	-0.0098	0.0007	0.0359
				(0.0107)	(0.0097)	(0.0262)	(0.0108)	(0.0101)	(0.0270)
Financial Depth							0.057	0.024	-0.0845
							(0.040)	(0.037)	(0.0996)
R-squared	0.12	0.43	0.25	0.17	0.43	0.32	0.23	0.44	0.34
Number countries	30	30	30	30	30	30	30	30	30

Standard errors reported in parentheses. GDP data corresponds to year 2006 and is measured in billions of US dollards (taken from World Bank databank). Average years of schooling corresponds to the population aged 15 or above in year 2005 (taken from the World Bank databank, using the Barro-Lee data). Financial depth is measured by the ratio of private credit by deposit money banks to GDP averaged during years 2004-06 (source: Global Financial Development Database). * significant 10%; ** significant 5%; *** significant 1%.

mechanism could be to do with different levels of financial development across economies.[26] In order to assess whether this is the reason behind the correlation patterns in Table 1, columns (6) - (9) of Table B.2 introduce a measure of financial depth (the ratio of total private credit by banks over GDP) as additional regressor. Once again, the main qualitative patterns exhibited by Table 1 remain unaltered, while the correlation coefficient associated to the ratio of firms doing process innovation becomes insignificant.

[26] See Crino and Ogliari (2017) for some recent evidence that financial markets are especially relevant for output quality upgrading.

Appendix C: A version of the model with continuous effort choice between labor and R&D

The benchmark model assumes that our two-member households are endowed with two discrete units of time: *i*) W's time endowment of which is *always* supplied as production labor; *ii*) R's time endowment, which may be consumed as leisure or, alternatively, be offered to firms as R&D effort. This ruled out the possibility of trade-offs between leisure and production effort. In addition, the $\{0, 1\}$ choice between R&D effort versus leisure generates a non-convexity in payoffs that translates households' choices into a simple comparison of bang-bang solutions. Here, we relax these two assumptions, and show how our main results in Section 3 and 4 continue to hold.

We let now $\varepsilon_l \in [0, 1]$ and $\varepsilon_{rd} \in [0, 1]$ denote the effort exerted by W and R in labor and R&D, respectively. R&D effort is given by $\varepsilon_{rd} \equiv \varepsilon_p + \varepsilon_q$, where ε_p and ε_q still denote process and product innovation effort, respectively. We keep assuming that type-R household members cannot set both $\varepsilon_p > 0$ and $\varepsilon_q > 0$ at the same time (i.e., they must specialize in one type of innovation effort). Henceforth, we replace the utility function in (4) by:

$$U_t = \ln \left[\sum\nolimits_{q \in Q_t} [q\, x(q)]^q \right] + \eta(1 - \varepsilon_{rd}) + \eta(1 - \varepsilon_l), \quad \text{with } \eta \geq 1. \tag{60}$$

The utility function (60) essentially adds the term $\eta(1 - \varepsilon_l)$ to (4).[27]

Given that innovation effort choices can now take any value within the unit interval, we need to amend a bit their effects relative to what was described in Assumption 1 and 2:

Process innovation effort: If R_i alive in t exerts $\varepsilon_{i,t,p} \in [0, 1]$ units of process innovation effort, and the state of nature in period t turns out to be $s(b^p_{i,t}) \in S$, then the process innovation blueprint created by R_i will boost the productivity of each unit of labor effort (in terms of the baseline quality variety output) by $\sigma\varepsilon_{i,t,p}$ units above that of the technology inherited at the beginning of t.

Product innovation effort: If R_i alive in t exerts $\varepsilon_{i,t,q}$ units of product innovation effort, and the state of nature in period t turns out to be $s(b^q_{i,t}) \in S$, then the product innovation blueprint created by R_i will allow the production of a new quality version of the good with $q_t = q_{t-1} + \rho\varepsilon_{i,t,q}$.

[27] The assumption $\eta \geq 1$ rather than $\eta > 0$ –as it was on (4)– is not crucial, and only placed to simplify the exposition by ensuring that the upper-bounds $\varepsilon_l \leq 1$ and $\varepsilon_{rd} \leq 1$ never bind in the optimum.

Growth via Process Innovation Effort

Let us first start again with the simplified framework where researchers can only exert process innovation effort. Like in the benchmark model, we will only focus on pure-strategy symmetric Nash equilibria. Consider a generic household i alive in period t. They will solve:

$$\max_{\varepsilon_{i,t,l}\in[0,1],\varepsilon_{i,t,p}\in[0,1]} : U_{i,t} = \ln\left(w_t\,\varepsilon_{i,t,l} + \varpi_t(\varepsilon_{i,t,p})\right) + \eta(1-\varepsilon_{i,t,p}) + \eta(1-\varepsilon_{i,t,l}),$$

where w_t denotes the wage per unit of labor effort in t and $\varpi_t(\varepsilon_{t,p})$ the salary paid to a researcher in exchange for $\varepsilon_{t,p}$ units of process innovation effort. Assuming that neither $\varepsilon_{i,t,l} \leq 1$ nor $\varepsilon_{i,t,p} \leq 1$ bind in the optimum, this yields as FOC:

$$\frac{\partial U_{i,t}}{\partial \varepsilon_{i,t,l}} = \frac{w_t}{w_t\,\varepsilon_{i,t,l} + \varpi_t(\varepsilon_{i,t,p})} - \eta \leq 0, \qquad \varepsilon_{i,t,l} \geq 0 \quad \text{and} \quad \frac{\partial U_t}{\partial \varepsilon_{i,t,l}} \cdot \varepsilon_{i,t,l} = 0; \tag{61}$$

$$\frac{\partial U_{i,t}}{\partial \varepsilon_{i,t,p}} = \frac{\varpi_t'(\varepsilon_{i,t,p})}{w_t\,\varepsilon_{i,t,l} + \varpi_t(\varepsilon_{i,t,p})} - \eta \leq 0, \qquad \varepsilon_{i,t,p} \geq 0 \quad \text{and} \quad \frac{\partial U_t}{\partial \varepsilon_{i,t,p}} \cdot \varepsilon_{i,t,p} = 0. \tag{62}$$

Note that the exact salary function $\varpi_t(\cdot)$ will be an equilibrium outcome, resulting from the aggregate behavior of all agents alive in t and satisfying the zero-profit condition for firms. In a symmetric Nash equilibrium where $\varepsilon_{j\neq i,t,l} = \varepsilon^*_{t,l}$, we will have:

$$\varpi_t^*(\varepsilon_{i,t,p}) = \sigma\,\varepsilon^*_{t,l}\,\varepsilon_{i,t,p}. \tag{63}$$

It can now be deduced from (63) that, given the unboundedness of the log utility function, there cannot exist an equilibrium where $\varepsilon^*_{t,l} = 0$ (as this would entail that households lifetime earnings equal zero). Secondly, replacing (63) into (62), and equalizing this to (61), it follows that an equilibrium with *positive* process innovation effort (i.e., $\varepsilon^*_{t,p} > 0$) must necessarily satisfy $\varpi_t'(\varepsilon^*_{t,p}) = \sigma\,\varepsilon^*_{t,l} = w_t$. This, in turn, means that:

$$\text{when } \varepsilon^*_{t,p} > 0 \quad \Rightarrow \quad \varepsilon^*_{t,l} = \frac{w_t}{\sigma}. \tag{64}$$

Plugging (63) and (64) into (61), we can finally obtain

$$\varepsilon^*_{t,p} = \max\left\{0, \frac{1}{\eta} - \frac{w_t}{\sigma}\right\}. \tag{65}$$

Like in the benchmark model, competition by firms for labor implies that the wage in t will be equal to its productivity when using the inherited technology. That is,

$$w_t = \begin{cases} 1 + \sigma \sum_{\tau=1}^{t-1} \varepsilon^*_{\tau,p} & \text{if } t \geq 2, \\ 1 & \text{if } t = 1. \end{cases} \tag{66}$$

Replacing (66) into (65), we thus obtain:

$$\varepsilon_{t,p}^{*}=\begin{cases}\max\left\{0,\dfrac{1}{\eta}-\dfrac{1}{\sigma}-\sum_{\tau=1}^{t-1}\varepsilon_{\tau,p}^{*}\right\} & \text{if } t\geq 2,\\ \max\left\{0,\dfrac{1}{\eta}-\dfrac{1}{\sigma}\right\} & \text{if } t=1.\end{cases} \tag{67}$$

We can now state a result analogous to the main result in Section 3.[28]

Result 1 *No single economy will be able to sustain positive process innovation effort forever. In particular:*
i) If $\sigma\leq\eta$, no process innovation ever takes place in equilibrium. Formally, $\varepsilon_{t,p}^{*}=0$ for all $t\geq 1$. This, in turn, implies that $Y_t=\eta^{-1}$ for all $t\geq 1$.
ii) If $\sigma>\eta$, the economy experiences positive process innovation effort only in $t=1$. Formally, $\varepsilon_{1,p}^{*}=\eta^{-1}-\sigma^{-1}>0$ and $\varepsilon_{t,p}^{*}=0$ for all $t\geq 2$. This, in turn, implies that $Y_1=\sigma^{-1}$ and $Y_t=\sigma\eta^{-2}$ for all $t\geq 2$.

Growth via Product Innovation Effort

In the benchmark model A_{t-1} in (1) denoted *number* of process innovations generated before t. Since $\varepsilon_{t,p}^{*}$ can take any value within $[0,1]$, we need to slightly amend this definition. Henceforth we introduce:

$$\mathcal{A}_{t-1}=\begin{cases}\sum_{\tau=1}^{t-1}\varepsilon_{t,p}^{*} & \text{if} \quad t\geq 2\\ 0 & \text{if} \quad t=1\end{cases}.$$

We now consider a generic period $t\geq 1$, assuming that no product innovation effort has taken place before t. When a generic R_i alive in t exerts $\varepsilon_{i,t,q}$ units of product innovation effort, he will create a blueprint that yield an expected return:

$$\pi_{i,t}^{q}=\left[P_t(\varepsilon_{i,t,q})\frac{1+\sigma\mathcal{A}_{t-1}}{1+\rho\varepsilon_{i,t,q}}-(1+\sigma\mathcal{A}_{t-1})\right]\varepsilon_{t,l}^{*}, \tag{68}$$

where

$$P_t(\varepsilon_{i,t,q})=(1+\rho\varepsilon_{i,t,q})\,Y_t^{\rho\varepsilon_{i,t,q}/(1+\rho\varepsilon_{i,t,q})}. \tag{69}$$

denotes the price that the firm will charge for each unit of the quality version $q_t=1+\rho\varepsilon_{i,t,q}$, should i's blueprint turn out to be the successful one.

Using (68) and (69), in a (symmetric) Nash equilibrium where all type-R household members alive in t exert $\varepsilon_{t,q}$ units of product innovation effort, Y_t will be given by:

$$Y_t=\left[\varepsilon_{t,l}\ (1+\sigma\mathcal{A}_{t-1})\right]^{1+\rho\varepsilon_{t,q}}. \tag{70}$$

[28] Note that when the equilibrium encompasses $\varepsilon_{t,p}^{*}=0$, condition (61) leads to $\varepsilon_{t,l}^{*}=\eta^{-1}<1$.

The first result we can now obtain is that there cannot exist an equilibrium with positive product innovation effort when $\mathcal{A}_{t-1} = 0$. Intuitively, when $\mathcal{A}_{t-1} = 0$, we will have that $\pi^q_{i,t} \leq 0$. Given that $\mathcal{A}_1 = 0$, this in turn means that no economy will ever exhibit an equilibrium with product innovation effort in $t = 1$.

Given the above result, the question to address now is whether economies satisfying $\sigma > \eta$ will be able to sustain an equilibrium with positive product innovation effort in $t = 2$.

Result 2 *Consider an economy that satisfies the condition $\sigma > \eta$. This economy will be able to sustain an equilibrium in $t = 2$ with positive product innovation effort if and only if $\sigma > \eta^2 \exp(\eta)$. In particular, when $\sigma > \eta^2 \exp(\eta)$, there exists an equilibrium in which all households set $\varepsilon^*_{2,q} > 0$. The exact level of $\varepsilon^*_{2,q}$ is implicitly pinned down by*

$$\frac{1 + \rho\varepsilon^*_{2,q}}{\exp(\eta\rho\varepsilon^*_{2,q})} = \frac{\eta^2}{\sigma}\exp(\eta), \tag{71}$$

which entails that $\varepsilon^*_{2,q}$ is increasing in σ. Moreover, in such and equilibrium $\varepsilon^*_{2,l} = \sigma^{-1}\eta\exp(\eta)$ and $Y^*_2 = \exp\left[\eta(1 + \rho\varepsilon^*_{2,q})\right]$.

Proof. Consider a generic household i alive in t. In an equilibrium with product innovation $\varpi_t(\varepsilon_{t,p}) = \pi^q_t$. Therefore, using (68) and (69), we obtain:

$$Y_{i,t}\left(\varepsilon_{i,t,q}, \varepsilon_{i,t,l}\right) = (1 + \sigma\mathcal{A}_{t-1})\left[\left(Y_t^{\rho\varepsilon_{i,t,q}/(1+\rho\varepsilon_{i,t,q})} - 1\right)\varepsilon^*_{t,l} + \varepsilon_{i,t,l}\right]. \tag{72}$$

Recalling (60), and using (69), we can observe that (within a symmetric Nash equilibrium with $\varepsilon^*_{t,l}$ and $\varepsilon^*_{t,q}$), household i will choose $\varepsilon_{i,t,q}$ and $\varepsilon_{i,t,l}$ by solving:

$$\max_{\varepsilon_{i,t,q}\in[0,1],\varepsilon_{i,t,l}\in[0,1]} : U_{i,t} = (1+\rho\varepsilon^*_{t,q})\ln\left[\left(Y_t^{\frac{\rho\varepsilon_{i,t,q}}{1+\rho\varepsilon_{i,t,q}}} - 1\right)\varepsilon^*_{t,l} + \varepsilon_{i,t,l}\right] + \eta\left[(1-\varepsilon_{i,t,q}) + (1-\varepsilon_{i,t,l})\right] + \Phi, \tag{73}$$

where $\Phi \equiv (1+\rho\varepsilon^*_{t,q})\left[\ln(1+\sigma\mathcal{A}_{t-1}) - \ln Y_t^{\rho\varepsilon^*_{t,q}/(1+\rho\varepsilon^*_{t,q})}\right]$ is a constant from i's viewpoint. Problem (73) yields as FOC:

$$\frac{\partial U_{i.t}}{\partial\varepsilon_{i,t,l}} = \frac{(1+\rho\varepsilon^*_{t,q})}{(Y_t^{\rho\varepsilon_{i,t,q}/(1+\rho\varepsilon_{i,t,q})} - 1)\varepsilon^*_{t,l} + \varepsilon_{i,t,l}} - \eta \leq 0, \quad \varepsilon_{i,t,l} \geq 0 \quad \text{and} \quad \frac{\partial U_t}{\partial\varepsilon_{i,t,l}}\varepsilon_{i,t,l} = 0;$$

$$\frac{\partial U_{i,t}}{\partial\varepsilon_{i,t,q}} = \frac{Y_t^{\rho\varepsilon_{i,t,q}/(1+\rho\varepsilon_{i,t,q})}\ \ln(Y_t)\ \varepsilon^*_{t,l}}{(1+\rho\varepsilon^*_{t,q})\left[\left(Y_t^{\frac{\rho\varepsilon_{i,t,q}}{1+\rho\varepsilon_{i,t,q}}} - 1\right)\varepsilon^*_{t,l} + \varepsilon_{i,t,l}\right]} - \eta \leq 0,\ \varepsilon_{i,t,q} \geq 0 \text{ and } \frac{\partial U_t}{\partial\varepsilon_{i,t,q}}\varepsilon_{i,t,q} = 0.$$

Recall we are focusing on symmetric Nash equilibrium. Thus, using (70) we can substitute $Y_t = \left[\varepsilon^*_{t,l}\,(1+\sigma\mathcal{A}_{t-1})\right]^{1+\rho\varepsilon^*_{t,q}}$, and solve for $\varepsilon^*_{i,t,q} = \varepsilon^*_{t,q}$ and $\varepsilon^*_{i,t,l} = \varepsilon^*_{t,l}$, which leads to:[29]

$$\frac{\partial U_t}{\partial \varepsilon_{t,l}} = \frac{(1+\rho\varepsilon^*_{t,q})}{\left(\varepsilon^*_{t,l}\right)^{1+\rho\varepsilon^*_{t,q}}(1+\sigma\mathcal{A}_{t-1})^{\rho\varepsilon^*_{t,q}}} - \eta = 0; \tag{74}$$

$$\frac{\partial U_t}{\partial \varepsilon_{t,q}} = \ln\left[\varepsilon^*_{t,l}\,(1+\sigma\mathcal{A}_{t-1})\right] - \eta \le 0, \quad \varepsilon_{t,q} \ge 0 \quad \text{and} \quad \frac{\partial U_t}{\partial \varepsilon_{t,q}}\varepsilon_{t,q} = 0. \tag{75}$$

Let us focus now on $t = 2$ when $\sigma > \eta$. Result 1 implies that in this case $\mathcal{A}_1 = \eta^{-1} - \sigma^{-1}$. Therefore, $1 + \sigma\mathcal{A}_{t-1} = \sigma/\eta$. Replacing this above, it follows that when $\sigma > \eta$ holds, (74) and (75) boil down to:

$$\frac{\partial U_2}{\partial \varepsilon_{2,l}} = \frac{(1+\rho\varepsilon^*_{2,q})}{\left(\varepsilon^*_{2,l}\right)^{1+\rho\varepsilon^*_{2,q}}\left(\dfrac{\sigma}{\eta}\right)^{\rho\varepsilon^*_{2,q}}} - \eta = 0; \tag{76}$$

$$\frac{\partial U_2}{\partial \varepsilon_{2,q}} = \ln\left(\varepsilon^*_{2,l}\,\frac{\sigma}{\eta}\right) - \eta \le 0, \quad \varepsilon_{2,q} \ge 0 \quad \text{and} \quad \frac{\partial U_2}{\partial \varepsilon_{2,q}}\varepsilon_{2,q} = 0. \tag{77}$$

Using (77), we can then observe that when $\varepsilon^*_{2,l} < \sigma^{-1}\eta\exp(\eta)$, the equilibrium will exhibit $\varepsilon^*_{2,q} = 0$. Then, replacing $\varepsilon^*_{2,q} = 0$ into (74) we obtain $\varepsilon^*_{2,l} = \eta^{-1}$. As a result, when $\sigma \le \eta^2\exp(\eta)$, in the equilibrium, $\varepsilon^*_{2,q} = 0$ will hold true.

Using again (77), we can see that for $\varepsilon^*_{2,q} > 0$ to hold, it must be the case that $\varepsilon^*_{2,l} = \sigma^{-1}\eta\exp(\eta)$. Replacing this expression into (74) leads to the expression in (71). Lastly, let now $\phi(\varepsilon^*_{2,q}) \equiv (1+\rho\varepsilon^*_{2,q})e^{-\eta\rho\varepsilon^*_{2,q}}$ and notice that $\phi(0) = 1$ and $\phi'(\varepsilon^*_{2,q}) < 0$. Therefore, it follows that when $\sigma > \eta^2\exp(\eta)$ there is one value of $\varepsilon^*_{2,q} > 0$ that satisfies (71). Moreover, since $\phi'(\varepsilon^*_{2,q}) < 0$, $\varepsilon^*_{2,q}$ is non-decreasing in σ. Notice finally that the fact that $\sigma > \eta^2\exp(\eta)$, in turn implies that $\varepsilon^*_{2,l} = \sigma^{-1}\eta\exp(\eta) < 1$, satisfying thus the upper-bound constraint on $\varepsilon^*_{2,l}$. ■

Effect of Product Innovation on Process Innovation Effort

This final subsection illustrates how our previous findings in Section 4.4 extend to this alternative version of the model. We will restrict the analysis to the case of an economy where $\sigma > \eta^2\exp(\eta)$, and study the level of process innovation effort that would take place in $t = 3$, after an equilibrium in $t = 2$ with $\varepsilon^*_{2,q} > 0$. In short, this subsection shows that this will lead to a positive level of process innovation effort in $t = 3$. As in the benchmark model, the reason for this result rests on the positive effect that (past) product innovations entail on the incentives to further undertake process innovation effort.

[29] The FOC (74) is already explicitly taking into account that (due to the unboundedness of the log utility function) there cannot exist and equilibrium with $\varepsilon^*_{t,l} = 0$.

Consider a generic household i in $t = 3$ setting $\varepsilon_{i,3,l} \in [0,1]$ and $\varepsilon_{i,3,p} \in [0,1]$. They will receive as total income

$$Y_{i,3} = w_3\,\varepsilon_{i,3,l} + \varpi_3(\varepsilon_{i,3,p}) = \underbrace{P_3(q_2)\frac{1+\sigma\mathcal{A}_1}{1+\rho\varepsilon^*_{2,q}}}_{w_3}\,\varepsilon_{i,3,l} + \underbrace{\sigma\,\varepsilon^*_{3,l}\,\varepsilon_{i,3,p}}_{\varpi_3(\varepsilon_{i,3,p})}. \tag{78}$$

Combining (78) with (60), it follows that i will choose $\varepsilon_{i,3,l}$ and $\varepsilon_{i,3,p}$ by solving:

$$\max_{\varepsilon_{i,3,l}\in[0,1],\varepsilon_{i,3,p}\in[0,1]} : U_{i,3} = (1+\rho\varepsilon^*_{2,q})\ln\left[(1+\sigma\mathcal{A}_1)\,\varepsilon_{i,3,l} + \sigma\varepsilon^*_{3,l}\varepsilon_{i,3,p}\right] + \eta\left[(1-\varepsilon_{i,3,p}) + (1-\varepsilon_{i,3,l})\right].$$

This yields the following FOC:

$$\frac{\partial U_{i,t}}{\partial\varepsilon_{i,3,l}} = \frac{(1+\rho\varepsilon^*_{2,q})(1+\sigma\mathcal{A}_1)}{(1+\sigma\mathcal{A}_1)\,\varepsilon_{i,3,l} + \sigma\varepsilon^*_{3,l}\,\varepsilon_{i,3,p}} - \eta \le 0, \quad \varepsilon_{i,3,l} \ge 0 \quad \text{and} \quad \frac{\partial U_t}{\partial\varepsilon_{i,3,l}}\cdot\varepsilon_{i,3,l} = 0; \tag{79}$$

$$\frac{\partial U_{i,t}}{\partial\varepsilon_{i,3,p}} = \frac{(1+\rho\varepsilon^*_{2,q})\sigma\varepsilon^*_{3,l}}{(1+\sigma\mathcal{A}_1)\,\varepsilon_{i,3,l} + \sigma\varepsilon^*_{3,l}\,\varepsilon_{i,3,p}} - \eta \le 0, \quad \varepsilon_{i,3,p} \ge 0 \quad \text{and} \quad \frac{\partial U_t}{\partial\varepsilon_{i,3,p}}\cdot\varepsilon_{i,3,p} = 0. \tag{80}$$

It is easy to show again that in a symmetric Nash equilibrium it must be the case that $\varepsilon^*_{2,q} > 0$, therefore (79) must hold with strict equality. In turn, this implies that for an equilibrium in $t = 3$ to encompass $\varepsilon^*_{3,p} > 0$, it must be that $\sigma\varepsilon^*_{3,l} = (1+\sigma\mathcal{A}_1)$. Hence, using the fact that when $\sigma > \eta^2\exp(\eta)$, we have $\mathcal{A}_1 = \sigma/\eta$, we can observe that:

$$\text{when } \varepsilon^*_{3,p} > 0 \quad \Rightarrow \quad \varepsilon^*_{3,l} = \frac{1}{\eta}. \tag{81}$$

Lastly, plugging (81), together with $\sigma\varepsilon^*_{3,l} = (1+\sigma\mathcal{A}_1)$, into (80), and letting this condition hold with strict equality, leads to the following result.

Result 3 *Consider an economy that satisfies $\sigma > \eta^2\exp(\eta)$ and that in $t = 2$ has exhibited an equilibrium with $\varepsilon^*_{2,q} > 0$, where $\varepsilon^*_{2,q}$ was determined by condition (71). Then, this economy will generate enough incentives to sustain an equilibrium in $t = 3$ with positive process innovation effort. In such an equilibrium,*

$$\varepsilon^*_{3,p} = \frac{\rho\varepsilon^*_{2,q}}{\eta}.$$

Proof. Suppose first that in equilibrium $\varepsilon_{i,3,p} = 0$. Plugging this into (79), yields $\varepsilon_{i,3,l} = (1+\rho\varepsilon^*_{2,q})/\eta$. Next, using $\varepsilon_{i,3,l} = \varepsilon^*_{3,l} = (1+\rho\varepsilon^*_{2,q})/\eta$ together with $\varepsilon_{i,3,p} = 0$ into (80), implies that, for this to be an optimum, $(1+\rho\varepsilon^*_{2,q}) \le 1$ must be verified. However, this cannot be true when $\varepsilon^*_{2,q} > 0$, hence it must be that the equilibrium will require $\varepsilon_{i,3,p} > 0$. Finally, to prove that $\varepsilon^*_{3,p} = \rho\varepsilon^*_{2,q}/\eta$, solve (80) as a strict equality, after plugging in $\varepsilon_{i,3,l} = \varepsilon^*_{3,l} = \eta^{-1}$ therein, and bearing in mind that $\mathcal{A}_1 = \eta^{-1} - \sigma^{-1}$. ■

Appendix D: Additional Proofs

Proof that the threshold $\underline{R}_2$ is a decreasing function of σ.
Note that an alternate way to implicitly define $\underline{A}_2$ is by applying logs on (21). This leads to:

$$\underbrace{(1+\rho)\ln\left(1+\sigma\underline{A}_2\right)-\ln\left(1+\sigma\underline{A}_2+\sigma\right)}_{\Psi(\rho,\sigma,\underline{A}_2)}=0. \tag{82}$$

From (82) we can observe several important properties of the function $\Psi(\rho,\sigma,\underline{A}_2)$: *i)* $\Psi'_{\underline{A}_2}(\cdot)>0$, *ii)* $\Psi'_{\rho}(\cdot)>0$, *iii)* $\Psi(\rho,\sigma,\underline{A}_2=0)=-\ln(1+\sigma)<0$, *iv)* $\lim_{\underline{A}_2\to\infty}\Psi(\rho=0,\sigma,\underline{A}_2)=0$. A first result to notice is that since $\Psi(\rho=0,\sigma,\underline{A}_2=\infty)=0$ for any $\sigma>0$, and we have that $\Psi'_{\rho}(\cdot)>0$, it must then be that for any $\rho>0$ there is a unique, finite and strictly positive value of $\underline{A}_2$ such that it satisfies $\Psi(\rho,\sigma,\underline{A}_2)=0$. Furthermore, combining this with $\Psi'_{\underline{A}_2}(\cdot)>0$, it follows that $\partial\underline{A}_2/\partial\rho<0$. Using now the full expression $\Psi'_{\rho}(\cdot)=\ln\left(1+\sigma\underline{A}_2\right)$, we can also see that $\partial\Psi'_{\rho}(\cdot)/\partial\sigma>0$. Since $\Psi(\rho=0,\sigma,\underline{A}_2=\infty)=0$ for any $\sigma>0$, and $\Psi'_{\rho}(\cdot)>0$, it must then be the case that, considering two generic $\underline{\sigma}<\overline{\sigma}$ and letting $\Psi(\rho,\underline{\sigma},\underline{A}_2\left(\underline{\sigma}\right))=0$ and $\Psi(\rho,\overline{\sigma},\underline{A}_2(\overline{\sigma}))=0$, we must have $\underline{A}_2(\overline{\sigma})<\underline{A}_2\left(\underline{\sigma}\right)$. ■

Proof of Existence of $\widehat{T}(\boldsymbol{\alpha}_T)$. Let first define $\Lambda(\cdot)\equiv\Gamma(\cdot)-\Upsilon(\cdot)$. Thus,

$$\Lambda(T)\equiv\frac{1+(1-\alpha)\,\rho T+\rho}{1+(1-\alpha)\,\rho T}-\frac{\ln(1+\alpha\sigma T+\sigma)}{\ln(1+\alpha\sigma T)}, \tag{83}$$

and notice that $\lim_{T\to 0}\Lambda(T)=-\infty$ and $\lim_{T\to\infty}\Lambda(T)=0$. Differentiating (83) with respect to T yields:

$$\Lambda'(T)=-\frac{\rho^2\,(1-\alpha)}{\left[1+(1-\alpha)\,\rho T\right]^2}-\alpha\sigma\frac{\dfrac{\ln(1+\alpha\sigma T)}{1+\alpha\sigma T+\sigma}-\dfrac{\ln(1+\alpha\sigma T+\sigma)}{1+\alpha\sigma T}}{\left[\ln(1+\alpha\sigma T)\right]^2}. \tag{84}$$

Computing the limits of (84) as $T\to 0$ and $T\to\infty$ yields, respectively: $\lim_{T\to 0}\Lambda'(T)=+\infty$ and $\lim_{T\to\infty}\Lambda'(T)=0$. Hence, the function $\Lambda(\cdot)$ approaches asymptotically $\Lambda(T)=0$ as $T\to\infty$, while $\Lambda(\cdot)<0$ for sufficiently low levels of T. Next, we state and prove two auxiliary results to build the proof that $\Lambda(\cdot)$ will necessarily become positive for some finite level of $T=\widehat{T}>0$, and remain positive for $T>\widehat{T}$, approaching thus $\Lambda(T)=0$ as $T\to\infty$ from above.

Lemma A1 *There exists a positive and finite value $T=\widetilde{T}$ such that $\Lambda'(\widetilde{T})=0$.*

Proof. *Equalizing the expression in (84) to zero leads to*

$$\Psi(T)\equiv\frac{\dfrac{\ln(1+\alpha T\sigma+\sigma)}{1+\alpha T\sigma}-\dfrac{\ln(1+\alpha T\sigma)}{1+\alpha T\sigma+\sigma}}{\left[\ln(1+\alpha T\sigma)\right]^2}\left[1+(1-\alpha)\,T\rho\right]^2=\frac{\rho^2\,(1-\alpha)}{\alpha\sigma}. \tag{85}$$

From the LHS of (85) notice first that $\lim_{T\to 0}\Psi(T)=\infty$. *Moreover, computing* $\lim_{T\to\infty}\Psi(T)$*:*

$$\lim_{T\to\infty}\Psi(T)=\underbrace{\lim_{T\to\infty}\left[\frac{1}{\ln(1+\alpha T\sigma)}\right]}_{\text{converges to }0}\times\underbrace{\lim_{T\to\infty}\left[\frac{\ln(1+\alpha T\sigma+\sigma)}{\ln(1+\alpha T\sigma)}\frac{\left[1+(1-\alpha)\,T\rho\right]^2}{(1+\alpha T\sigma)}-\frac{\left[1+(1-\alpha)\,T\rho\right]^2}{(1+\alpha T\sigma+\sigma)}\right]}_{\text{converges to }\frac{(1-\alpha)^2\rho^2}{\alpha^2\sigma}}=0.$$

Therefore, since $\frac{\rho^2(1-\alpha)}{\alpha\sigma}>0$, *by continuity there must then exist some positive and finite value* $T=\widetilde{T}$ *such that* $\Lambda'(\widetilde{T})=0$. ♦

Lemma A2 For T sufficiently large $\Lambda(T)>0$. In other words, $\lim_{T\to\infty}\Lambda(T)=0^+$.

Proof. Note from (83) that $\Lambda(T)>0$ if and only if

$$\frac{(1+\alpha\sigma T)^{1+(1-\alpha)\rho T+\rho}}{(1+\alpha\sigma T+\sigma)^{1+(1-\alpha)\rho T}}>1. \tag{86}$$

Applying the limit as $T\to\infty$ on the RHS of (86):

$$\lim_{T\to\infty}\left[\frac{(1+\alpha\sigma T)^{1+(1-\alpha)\rho T+\rho}}{(1+\alpha\sigma T+\sigma)^{1+(1-\alpha)\rho T}}\right]=\lim_{T\to\infty}\left[\frac{(1+\alpha\sigma T)^{1+(1-\alpha)\rho T}}{(1+\alpha\sigma T+\sigma)^{1+(1-\alpha)\rho T}}\right]\times\lim_{T\to\infty}(1+\alpha\sigma T)^{\rho}.$$

Note first that $\lim_{T\to\infty}(1+\alpha\sigma T)^{\rho}=+\infty$. Secondly, let

$$\Xi\equiv\frac{(1+\alpha\sigma T)^{1+(1-\alpha)\rho T}}{(1+\alpha\sigma T+\sigma)^{1+(1-\alpha)\rho T}}. \tag{87}$$

Applying logs on the RHS of (87), it follows that:

$$\lim_{T\to\infty}\ln(\Xi)=\lim_{T\to\infty}\frac{\ln\left(\dfrac{1+\alpha\sigma T}{1+\alpha\sigma T+\sigma}\right)}{\left[1+(1-\alpha)\,\rho T\right]^{-1}}. \tag{88}$$

Next, using L'Hopital's rule to compute the limit in the RHS of (88) leads to:

$$\lim_{T\to\infty}\ln(\Xi)=-\frac{1-\alpha}{\alpha}\rho. \tag{89}$$

Hence, using finally the fact that $\lim_{T\to\infty}\ln(\Xi)=\ln\left[\lim_{T\to\infty}(\Xi)\right]$, it follows that $\lim_{T\to\infty}(\Xi)=e^{-\frac{1-\alpha}{\alpha}\rho}>0$. Therefore, it must be the case that (86) holds true for a sufficiently large value of T and $\lim_{T\to\infty}\Lambda(T)=0^+$. ♦

Merging the results of the previous two lemmas, plus the fact that $\lim_{T\to 0}\Lambda(T)=-\infty$ and $\lim_{T\to\infty}\Lambda(T)=0$, by continuity it follows there must exist some finite positive value $\widehat{T}<\widetilde{T}$, such that: *i)* $\Lambda(T)<0$ for all $0<T<\widehat{T}$, *ii)* $\Lambda(\widehat{T})=0$, and *iii)* $\Lambda(T)<0$ for all $T>\widehat{T}$. ■

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